

RUSSIAN MINISTRY OF SCIENCE AND EDUCATION
FEDERAL STATE BUDGETARY EDUCATIONAL INSTITUTION
OF HIGHER EDUCATION
«BASHKIR STATE UNIVERSITY»

FACULTY OF MATHEMATICS AND INFORMATION TECHNOLOGIES

Approved: at the department meeting
Protocol # 5 from February 28, 2022
Head of the department

Coordinated with:
EMC chairman of the faculty/institute



_____ Z. Yu. Fazullin



_____ A.M. Efimov

WORKING PROGRAM OF DISCIPLINE (MODULE)

Discipline Analysis II

(name of the discipline)

Obligatory part

(name of the part enclosing the discipline (obligatory, formed by participants of the educational activity, facultative))

bachelor (undergraduate) program

Course of training (speciality)

01.03.02 Applied mathematics and informatics

(code and name of the course of training (speciality))

Subdivision of the course of training (profile)

Applied programming and data analysis

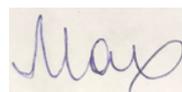
(name of the profile of training)

Qualification (level of training)

bachelor

(name of the level of training)

Designer (compiler):
associate professor of the MA
department, PhD



_____ A.A. Makhota

For enrollment of: 2021

Ufa 2022

Designer: associate professor, PhD Alla Aleksandrovna Makhota

The working program of the discipline is approved at the meeting of the department of Mathematical Analysis,
Protocol # 5 from February 28, 2022

Head of the department



Z. Yu. Fazullin

The addenda and updates introduced into the working program of the discipline are approved at the meeting of the department of Higher algebra and geometry,
protocol # 11 from June, « 10 » 2022.

Head of the department



Z. Yu. Fazullin

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1. The list of planned learning outcomes of the discipline, correlated with the planned results of the educational program

Mastering the discipline must lead to forming of the following competence (GPC-1):

Category (group) of competencies (if there is exist a GPC)	Competence to be formed (with the code)	Code and name of the indicator of competence achievement	Learning outcomes for the discipline
<i>Theoretical and practical basics of professional activity</i>	<i>GPC-1. Able to apply the fundamental knowledge obtained in the mathematical and/or natural sciences and use them in professional activities</i>	<i>GPC-1.1. Possesses basic knowledge obtained in the field of mathematical analysis.</i>	<i>Know the basic concepts of disciplines, basic theorems and consequences, methods of solution and analysis of typical problems</i>
		<i>GPC-1.2. Is able to use them in professional activities.</i>	<i>Be able to use in practice the knowledge of disciplines, correctly formulate tasks and reasonably choose methods of their solutions</i>
		<i>GPC-1.3. Has the skills to choose the methods of solving the problems of professional based on theoretical knowledge.</i>	<i>Master the basic mathematical knowledge and its application to the solution of problems theoretical and applied character</i>

2. The discipline "Mathematical Analysis" is part of the basic part. The discipline is studied at the 1. 2 courses in semesters 1-3. "Complex analysis" is a continuation of mathematical analysis and the discipline is studied in the 4th semester

The purpose of the discipline: studying the basics of mathematical analysis, combining the theory of real numbers, the theory of limits, the theory of series, differential and integral calculus and their direct applications; development of the ability to understand and apply in the research and applied activities of the modern mathematical apparatus; development of logical, heuristic and algorithmic thinking.

To master the discipline, competencies formed in the study of school mathematics course are necessary. The discipline is closely related to such disciplines as "Complex analysis", "Applied Functional Analysis", "Differential Geometry", "Differential Equations", "Probability Theory and Mathematical Statistics", "Optimization Methods", "Equations with Partial Derivatives".

The purpose of the discipline "Complex analysis" is: obtaining knowledge in the field of functions of the complex variable, the fundamental training of students in the theory of functions in the complex area, mastering the methods of solving the basic problems of the theory of functions of the complex variable, mastering the modern mathematical apparatus for further use in the study of mathematical disciplines and applications. To master the discipline requires competence formed in the study of the following disciplines: "Mathematical Analysis", "Algebra", "Differential equation".

Mastering the discipline "Complex analysis" is necessary for the subsequent study of disciplines "Functional analysis", "Equations in partial derivatives", "Numerical methods" and several others.

The discipline is closely connected with such disciplines as "Higher Algebra", "Mathematical Analysis", "Functional Analysis".

3. The content of the working program (the volume of the discipline, types and types of classes, educational and methodological support of independent work of students)

The content of the working program is presented in Appendix № 1.

4. Evaluation funds for the discipline

4.1. List of competencies and indicators of achievement of competencies with the planned learning outcomes related to them in the discipline. Description of criteria and scales for evaluating the results of training in the discipline.

Code and definition of each competence.

GPC-1: Ability of applying the fundamental knowledge gained in the field of mathematical and (or) natural sciences and using them in professional activities.

Level of competence acquirement	Planned learning outcomes (indicators of achievement of a predetermined level of acquired competencies)	Evaluation criteria for the results			
		2 (" Non-satisfactory")	3 ("Satisfactory")	4 ("Good")	5 ("Excellent")

First level	To know: basic concepts of disciplines, basic theorems and their consequences, methods of solving and analyzing typical problems	Lack of knowledge	Incomplete ideas about the basic concepts of the discipline, the main theorems and their consequences, methods of solving and analyzing typical problems	Formed knowledge and skills, but containing separate gaps in the basic concepts of the discipline, the main theorems and their consequences, methods of solving and analyzing typical problems	Formed systematic knowledge and skills on the basic concepts of the discipline, the main theorems and their consequences, methods of solving and analyzing typical problems
Second level	Be able to: use the knowledge of the disciplines in practice, correctly formulate tasks and reasonably choose methods of their solution	Lack of skills	Partial skills of using the knowledge of the disciplines in practice, correctly formulate tasks and reasonably choose methods of their solution	Generally formed skills, but containing some gaps in the ability of using knowledge of the disciplines in practice, correctly formulate tasks and reasonably choose methods of their solution	Well-formed ability using the knowledge of the disciplines in practice, correctly formulate specific tasks and reasonably choose methods of their solution
Third level	Be in possession of: basic mathematical knowledge and its application for solving theoretical and applied problems	Lack of formed skills	Generally successful, but non-systematic application of basic mathematical knowledge for solving theoretical and applied problems	Generally successful, but containing some gaps in the application of basic mathematical knowledge for solving theoretical and applied problems	Successful acquisition of basic mathematical knowledge and its application for solving theoretical and applied problems

Level of competence acquirement	Planned learning outcomes (indicators of achievement of a predetermined level of acquired competencies)	Evaluation criteria for the results (pass/fail rating)	
		Grade of "non pass" ("failed")	Grade of "pass" ("passed")
First level	To know: basic concepts of the disciplines, basic theorems and their consequences, methods of solving and analyzing typical problems	Incomplete ideas about the basic concepts of the discipline, the main theorems and their consequences, methods of solving and analyzing typical problems	Well-formed systematic ideas about the main concepts of the discipline, the main theorems and their consequences, methods of solving and analyzing typical problems
Second level	Be able to: use the knowledge of the disciplines in practice, correctly formulate tasks and reasonably choose methods of their solution	Partial skills of using the knowledge of the disciplines in practice, correctly formulate tasks and reasonably choose methods of their solution	Well-formed ability of using the knowledge of the disciplines in practice, correctly formulate tasks and reasonably choose methods of their solution
Third level	Be in possession of: basic mathematical knowledge and its application for solving theoretical and applied problems	Generally successful, but non systematic application of basic mathematical knowledge for solving theoretical and applied problems	Successful acquirement of basic mathematical knowledge and its application for solving theoretical and applied problems

Evaluation criteria are the points which are given by the teacher to each student for different types of activities (evaluation tools) at the end of the study of each module (section of a discipline) that are listed in the rating plan of the discipline (for the exam: current control –40 points maximum; periodical control – 30 points maximum, bonus points – 10 points maximum. For the grade of “pass”: current control - 50 points maximum; periodical control – 50 points maximum, bonus points – 10 points maximum).

Grading scale:

(for the exam:

45 to 59 points – "satisfactory";

60 to 79 points – "good";

80 points – "excellent".

for the grade of “pass”:

“passed” - **from 60 to 110** rating points (including 10 bonus points),

“failed” - **from 0 to 59** rating points).

4.2. Standard control tasks or other materials necessary for evaluating the results of training in the discipline, correlated with the indicators of competence achievement established in the educational program. Methodological materials defining the procedures for evaluating the results of training in the discipline

Indicator of the achieved competence	Results of training in the discipline	Evaluation tools
GPC-1. 1. Possesses basic knowledge gained in the field of mathematical analysis	To know: the basic concepts of the discipline, the main theorems and their consequences, methods of solving and analyzing typical problems	Theoretical survey, exam
GPC-1. 2. Knows how to use them in professional activities	Be able to: use the knowledge of the discipline in practice, correctly formulate tasks and reasonably choose methods of their solution	Laboratory work, test work, pass/fail rating
GPC-1.3. Possesses the skills to choose methods of solving problems in professional activity based on theoretical knowledge.	To possess: basic mathematical knowledge and its application for solving theoretical and applied problems	Laboratory work, test work, pass/fail rating

Examination tickets

Exam and credit is an assessment tool for all stages of mastering the competencies.

Questions for the exam (third semester):

1. The concept of a functional sequence and a functional series. Convergence of functional series at a point and on a set. Uniform convergence. The Cauchy criterion for uniform convergence.
2. Power series. The Cauchy-Adamar theorem. Definition of the coefficients of power series (Taylor's series). Taylor's series expansion of elementary functions.
3. Curvilinear integrals of the first kind. Reduction to a regular definite integral.
4. Curvilinear integrals of the second kind. Its existence and computation.
5. Calculation of areas by means of the curvilinear integral.
6. Conditions for the independence of the curvilinear integral from the path of integration.
7. The test of the exact differential and finding the first form in the case of a rectangular area.
8. The definition of the double integral. The problem of the volume of a cylindrical bar. The reduction of the double integral to the repeated integral.
9. Green's formula.

10. Substitution of variables in a double integral.
11. Expression of area in curvilinear coordinates.
12. The concept of surface. Types of surface definition: explicitly, implicitly, parametrically.
13. Tangent plane and normal to the surface.
14. Side of the surface. Orientation of the surface in space.
15. Definition and calculation of surface area.
16. Definition of the surface integral. Reduction to the double integral.
17. The expression of the volume of a body by the surface integral.
18. Definition of the triple integral and the conditions for its existence.
19. Calculation of the triple integral extended to the parallelepiped.
20. Ostrogradsky's formula.
21. Substitution of variables in triple integrals. Transformation of spaces and curvilinear coordinates.
22. Expression of volume in curvilinear coordinates.

Questions for the exam (fourth semester):

1. Complex numbers and operations on them. The complex plane.
2. The limit of a sequence. Functions of a complex variable. Limits of a function of a complex variable. Continuity.
3. Differentiability (definition, properties) Differentiability and Cauchy-Riemann conditions.
4. Harmonic functions. Reconstructing an analytic function by its real or imaginary part.
5. Function $\sqrt[n]{z}$.
6. Function e^z . The logarithmic function.
7. Elementary functions (power, natural exponent, general power and general exponent functions). Trigonometric functions.
8. Zhukovsky's function.
9. Fractional and linear functions.
10. Integral of a function of a complex variable. The simplest properties.
11. Cauchy's integral theorem.
12. Decomposition of analytic function in Taylor series.
13. Laurent series. The expansion of functions into Laurent series (Loran theorem).
14. Isolated singular points. Classification and examples.
15. Isolated singular points of unambiguous character. Relation between the main part of Laurent series and the type of singularity.

16. Residue (definition, examples). Residue at an infinitely distant point.
17. Basic theorems about residues. (Cauchy theorem about residues, Theorem about the sum of residues).
18. Ways of calculating residues at poles.
19. Application of residues theory to the computation of definite integrals

$$\int_0^{2\pi} R(\cos\phi, \sin\phi) d\phi$$

20. Applications of the residues theory to the calculation of integrals

$$\int_{-\infty}^{\infty} \frac{P(x)}{Q(x)} dx.$$

21. Application of residues Theory to the Calculation of Definite Integrals Jordan's lemma.

$$\int_{-\infty}^{\infty} f(x) e^{i\lambda x} dx.$$

Structure of the examination ticket:

1. Theoretical question.
2. Theoretical question

Grading criteria (in points):

- **25-30 points** if student demonstrates the knowledge of 80% or more of the required educational material in the discipline.
- **17-24 points** if student demonstrates the knowledge from 60% to 79% of the required educational material in the discipline.
- **10-16 points** if student demonstrates the knowledge from 45% to 59% of the required educational material in the discipline.
- **1-10 points** if student demonstrates the knowledge less than 45% of the required educational material in the discipline.

An example of examination tickets is given in Appendix 2.

Sample questions for verbal examination (Semester 3)

Module 1.

1. The concept of a functional sequence (series).
2. Convergence of functional series at a point and on a set.
3. Uniform convergence of a functional sequence (series).
4. Cauchy criterion for uniform convergence of a functional sequence (series).
5. Weierstrass criterion for uniform convergence of functional series.
6. Abel tests.
7. Dirichlet's tests.

Module 2.

1. Definition of power series.
2. Abel theorem.
3. Cauchy-Adamar theorem.
4. The radius of convergence of a power series.
5. Decomposition into a power series at a point of a function

Module 3.

1. Curvilinear integrals of the first kind.
2. Reduction of the curvilinear integral of the first kind to the usual definite integral.
3. Curvilinear integrals of the second kind.
4. Calculation of area using the curvilinear integral.
5. Conditions for the independence of the curvilinear integral from the path of integration.
6. The closed curve.
7. A curve without self-intersections.
8. A smooth curve.

Module 4.

1. Definition of the double integral.
2. The problem of the volume of a cylindrical bar.
3. The reduction of the double integral to the repeated integral.
4. Green's formula.
5. Substitution of variables in the double integral.
6. Expression of area in curvilinear coordinates.
7. The concept of surface.
8. Tangent plane and normal to the surface.

Sample questions for verbal examination (Semester 4)

Module 1..

1. Complex numbers and operations on them. The complex plane.
2. Elementary functions of complex variable.
3. The limit of a sequence.
4. Cauchy-Riemann conditions.
5. Harmonic functions.

Module 2..

1. Integral of a function of a complex variable. The simplest properties.
2. Cauchy's integral theorem.
3. Decomposition of analytic function in Taylor series.

Module 3..

1. Laurent series. The expansion of functions into Laurent series (Loran theorem).
2. Isolated singular points. Classification and examples.
3. Isolated singular points of unambiguous character. Relation between the main part of the Laurent series and the type of singularity.
4. Residue (definition, examples). Basic theorems about residues.
5. Application of residues theory to the computation of definite integrals

Each student is given one question from each module during the semester.

Each question is graded 2 points. These points are taken into account when calculating the rating only for the exam.

Grading criteria (in points):

- 15 points are awarded to a student if he/she gives a clear, detailed and correct answer to the question;

- 10-14 points are awarded if the student gave a correct answer, but may be incomplete or did not answer 1-2 additional questions;

- 1-9 points are awarded if the student gave an incorrect answer to the question, but answered additional questions or gave a fragmented, but correct answer to the main question, but did not answer additional questions;

- 0 points will be assigned to the student if he or she fails to answer the main question and does not answer any additional questions.

Work in class

Class work: solving problems from the book [3] (in 3rd semester) and [5] (in 4th semester). Each activity is graded 1 point.

Assignments for laboratory work

Each lab work consists of several problems on the topic being tested. Each student is given a separate version. All laboratory works are performed by each student independently. The problem is considered solved correctly if the correct solution is written in detail and clearly. Upon completion of the laboratory work, the student submits a report that contains the solutions to the problems on the given laboratory work.

Examples of a variant of the laboratory work:

Calculate and represent on the complex plane

1. $\left(\frac{1-2i}{2+i}\right)^6$;
2. $\left(\frac{3i-2}{2+3i}\right)^{10}$;

3. $\left(\frac{\sqrt{2}-i}{\sqrt{3}+i}\right)^{15}$;
4. $\left[(\sqrt{3}-i)(1+i\sqrt{3})\right]^8$;
5. $\left(\frac{-2}{1+i}\right)^{14}$;
6. $\frac{1}{2}(7i-5)^{10}$;
7. $\left(\frac{(3+4i)(1-2i)}{i}\right)^4$;
8. $\left(\frac{1-i\sqrt{3}}{1+i\sqrt{3}}\right)^3$;
9. $(\sin 30^\circ + i \sin 60^\circ)^3$;
10. $\left(\frac{-2-2i}{1+2i}\right)^{10}$;
11. $(1+i)^8$;

Description of the evaluation methodology for the test works:

Grading Criteria (in points): The maximum number of points a student can earn for a correctly solved problem is written in front of each problem. For solving the problem you can get 0.5, 1, 1.5, 2 or 3 points depending on the correctness of the solution given (there can be inaccuracies in the solution, incomplete solution, not completely correct solution).

Examples of a variant of the test work:

Calculate integrals using residues:

$$1. \int_{-\infty}^{+\infty} \frac{1}{(x^2+1)} dx$$

$$2. \int_0^{2\pi} \frac{\sin^2 x}{(3+2 \cos x)} dx$$

$$3. \int_0^{\infty} \frac{\cos x}{(1+x^4)} dx$$

$$4. \int_{|z|=1} \frac{\operatorname{ctg} z}{z} dx$$

Each task is graded 3 points.

Calculating and graphic work

Problem 1: Find the modulus and principal value of an argument of a complex number $z = -\sqrt{3} - i$.

Problem 2: Write a complex number in exponential form $z = 2 + 2\sqrt{3}i$.

Problem 3: Find all value of roots $\sqrt[4]{\frac{-1-i\sqrt{3}}{2}}$.

Problem 4: Raising to a power $(3-i\sqrt{3})^6$.

Problem 5: Find out the geometrical meaning of the relation $|z-i|=3$.

Problem 6: Let $f(z) = z\bar{z}$, $g(z) = z^2e^z$, $\phi(z) = x^2 - y^2 + 2ixy$, $p(z) = x^2 - y^2 - 2ixy$, $h(z) = \bar{z}$, where $z = x+iy$, $\bar{z} = x-iy$. Find out which of these functions are analytical.

Problem 7: Let imagine part $v = \frac{x}{x^2 + y^2}$, find real part $u(x, y)$ of analytical function

$$f(z) = u(x, y) + iv(x, y), \quad z = x + iy.$$

4.3. Rating – plan of the discipline

Rating-plan of the discipline is given in Appendix 3.

5. Educational, methodic and informational support of the discipline.

5.1. List of references to primary and complementary educational literature necessary for acquiring the discipline.

Primary literature

1. Fichtenholz G.M. Course of differential and integral calculus. In 3 vols. M.: Fizmatlit, 2001. (in Russian) http://biblioclub.ru/index.php?page=book_red&id=83037
2. William F. Trench Introduction to Real Analysis <https://digitalcommons.trinity.edu/mono/7/>
3. Demidovich B.P. Collection of Problems and Exercises in Mathematical Analysis: Textbook for Universities Moscow: CheRo, 1997. http://biblioclub.ru/index.php?page=book_red&id=459722
4. Башмаков Р.А., Махота А.А. «Введение в ТФКП». Уфа, РИЦ БашГУ, 2012.
5. Л.И.Волковыский, Г.Л.Лунц, И.Г.Арамонович «Сборник задач по теории функций комплексного переменного», М.: Физматлит, 2002.
6. Башмаков Р.А., Махота А.А. «Введение в комплексный анализ». Электронный учебник 2012 (свидетельство о регистрации электронного ресурса №18361 ИНИПИ РАО ОФЭРНиО).

Auxiliary literature:

1. Michael D. Alder, An Introduction to Complex Analysis for Engineers. 1997
<http://www.freebookcentre.net/maths-books-download/An-Introduction-to-Complex-Analysis-for-Engineers.html>
2. Шабат Б.В. Введение в комплексный анализ. М.: Государственное издательство физико-математической литературы, 1961.
http://biblioclub.ru/index.php?page=book_red&id=464254&sr=1

5.2. List of the Internet resources and software necessary for acquiring the discipline, including professional data bases and reference systems.

1. Library of Bashkir State University <http://lib.bashedu.ru>
2. BashSU Electronic Library System <https://elib.bashedu.ru>
3. University WebWork server: <http://webwork-okko.bashedu.ru/webwork2/>.

6. Hardware equipment, materials and rooms necessary for implementing the educational process in the discipline.

<i>Names of specialized rooms, rooms and laboratories</i>	<i>Activity form</i>	<i>Name of the equipment/software</i>
<i>1</i>	<i>2</i>	<i>3</i>
Rooms 501,517, 528 or any other room according to the current time table	<i>Lectures</i>	The board for writing, projector and screen
Rooms 517, 503 or any other room according to the current time table	<i>Laboratory/practical classes</i>	The board for writing, projector and screen
Library, reading halls	<i>Individual work</i>	Internet. The university WebWork server

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CONTENT OF THE WORKING PROGRAM

of the discipline Analysis II for semester 3

Activity	Duration
Total duration of the discipline (CUD / hours)	7/252
Academic hours for the work with instructor	
lectures	54
laboratory	54
practical classes	18
other (consultation in group or individually and other forms of learning activities assuming collaboration of learners with instructor)	1,7
Academic hours for individual work of learners	71,5
Academic hours for preparing to exam/credit test/differentiated credit test (Grading)	52,8
laboratory	

Final grading:

exam in semester 3

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CONTENT OF THE WORKING PROGRAM

of the discipline Analysis II for semester 4

Activity	Duration
Total duration of the discipline (CUD / hours)	4/144
Academic hours for the work with instructor	
lectures	32
laboratory	32
practical classes	32
other (consultation in group or individually and other forms of learning activities assuming collaboration of learners with instructor)	1,7
Academic hours for individual work of learners	2,5
Academic hours for preparing to exam/credit test/differentiated credit test (Grading)	43,8
laboratory	

Final grading:

exam in semester 4

Semester 3

Item no.	Topic and its content	Learning forms: lectures, seminars, laboratory, and individual work with duration (in academic hours)				Primary and auxiliary literature (numbers in the reference list)	Task for individual work of learners	Forms of current grading (colloquia, quizzes, computer tests etc)
		LEC	SEM	LAB	IND			
1	2	3	4	5	6	7	8	9
1.	Functional sequences and series, uniform convergence. Signs of uniform convergence. Theorem on limit transition. Theorems on Continuity, termwise Integration and differentiation.	10	4	8	12	1-3,	[3]: 2746-2754, 2770-2778	Theoretical survey Laboratory work
2.	Power series, radius of convergence, Cauchy - Adamar's formula. Uniform convergence and continuity of the sum of a power series.	8	4	8	12	1-3	[3]: 2813-2826, 2869-2873	Theoretical survey Laboratory work
3.	The double integral. The integrability criterion of Lebesgue and Darboux. The double integral, its geometrical interpretation and basic properties; the reduction the double integral to the iterated integral;	8	4	10	12	1-3	[3]: 3924-3929, 3948, 3951-3954	Laboratory work, verbal questioning
4.	Substitution of variables in the double integral; area surface; mechanical and physical applications of of double integrals; integrals of higher multiplicity; their definition, calculation,	10		10	12	1-3	[3]: 4008,4013, 4022, 4037,4052, 4062, 4076, 4081, 4102, 4107,4116	Theoretical survey Laboratory work

	and the simplest properties; improper and multiple integrals.							
5.	Curvilinear integrals and surface integrals; the Green's formula; the Ostrogradsky's formula; the Stokes formula; the conditions of independence of of the curvilinear integral from the path of integration.	10	6	10	12	1-3	[3]: 4221, 4224-4227, 4241, 4250-4253, 4259, 4296, 4308-4311	Laboratory work, verbal questioning
6.	Surface integrals. Elements of field theory. Vector interpretation of Ostragradsky's and Stokes formulas.	8		8	11,5	1-3	[3]: 4370, 4376, 4401-4403, 4452	Theoretical survey Laboratory work
	Total Hours	54	18	54	71,5			

Semester 4

Item no.	Topic and its content	Learning forms: lectures, seminars, laboratory, and individual work with duration (in academic hours)				Primary and auxiliary literature (numbers in the reference list)	Task for practical/individual work of learners	Forms of current grading (colloquia, quizzes, computer tests etc)
		LEC	SEM	LAB	IND			
1	2	3	4	5	6	7	8	9
1.	Complex numbers, the complex plane; modules and argument of a complex number, their properties. Numerical sequences and their limits, series; limit of a sequence of complex numbers. Cauchy's criterion. Infinitely distant point.	4	4	4		4-6 Aux.1,2	[5]: 5. 1.100-105	Theoretical survey, Laboratory work Verbal questioning
2.	Functions of a complex variable; Limit of a function; Continuity, differentiability on a complex variable	2	2			4-6 Aux.1,2	[5]: 5 1.120 5. 1.126	Verbal questioning
3.	Analytic function; Cauchy-Riemann condition; Harmonic functions. Reconstructing an analytic function by its real or imaginary part.	4	4	4		4-6 Aux.1,2	[5]: 5. 135 5. 1.167 5. 1.167 5.1.188	Laboratory work Verbal questioning Calculating and graphic work

4.	Elementary functions (power, natural exponent, general power and general exponent functions). Trigonometric functions. Fractional and linear functions.	2	2	2	2,5	4-6 Aux.1,2	[5]: 5. 2.154 5. 2.181	Theoretical survey, Laboratory work
5.	Zhukovsky's function.	2	2	2		4-6 Aux.1,2	[5]: 5. 2.108 5. 2.117	Verbal questioning
6.	Integral of a function of a complex variable. The simplest properties. Cauchy's integral theorem.	4	2	4		4-6 Aux.1,2	[5]: 5. 3.9 5. 3.20	Theoretical survey, Laboratory work Verbal questioning
7.	Complex numerical series. Convergence. Sequences and series of analytic functions in a domain.	2	2	2		4-6 Aux.1,2	[5]: 5. 1.96 5. 1.99 5. 5.9	Laboratory work Verbal questioning
8.	Decomposition of an analytic function into a power series (Taylor series of an analytic function).	2	2	2		4-6 Aux.1,2	[5]: 5. 3.74 5. 3.83	Theoretical survey, Laboratory work

9.	Laurent series, its convergence area; expansion of analytic function into Laurent series, uniqueness of expansion. Relation of the main part of the Laurent series to the character of a singular point	4	4	4		4-6 Aux.1,2		Theoretical survey, Laboratory work Verbal questioning
10.	Classification of isolated singular points of a single-valued analytic function. Pole, order of the pole;	2	2	2		4-6 Aux.1,2	[5]: 5. 4.71 5. 4.77	Laboratory work Verbal questioning
11.	Residues of an analytic function at an isolated singular point. Basic theorem of the theory of residue	2	4	4		4-6 Aux.1,2	[5]: 5. 4.113 5. 4.89 5. 4. 94 5. 4.102	Theoretical survey, Laboratory work
12.	Calculation of definite real integrals using residue	2	2	2		4-6 Aux.1,2	[5]: 5. 4.121 5. 4.124	Laboratory work
	Total hours:	32	32	32	2,5			

An example of examination tickets (semester 3)

FEDERAL STATE BUDGETARY EDUCATIONAL INSTITUTION
OF HIGHER EDUCATION
«BASHKIR STATE UNIVERSITY»
FACULTY OF MATHEMATICS AND INFORMATION TECHNOLOGIES
Department of Mathematical Analysis

EXAMINATION TICKET #1
on “Analysis II”

1. Curvilinear integrals of the first kind. Reduction to a regular definite integral.
2. Ostrogradsky’s formula.

An example of examination tickets (semester 4)

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Department of Mathematical Analysis

EXAMINATION TICKET #2
on “Analysis II”

1. Elementary functions of complex variable.
2. Basic theorems about residues. (Cauchy theorem about residues, Theorem about the sum of residues).

Rating – plan of the discipline

Analysis II

(the name of the discipline according to the working curriculum)

Direction 01.03.02 Applied Mathematics and Informatics

Grade 2, semester 3

Rating-plan No. 1 (exam)

Types of educational activities of students	Points for a specific task	Number of tasks per semester	Points	
			minimum	maximum
Module 1. Curvilinear integrals				
Current control			0	10
Classroom work	1	10	0	10
Periodical control			0	7,5
Theoretical survey (inquiry)	2,5	3	0	7,5
Module 2. Power series				
Current control			0	10
Classroom work	1	10	0	10
Periodical control			0	7,5
Theoretical survey (inquiry)	2,5	3	0	7,5
Module 3. Multiple Integrals.				
Current control			0	10
Classroom work	1	10	0	10
Periodical control			0	7,5
Theoretical survey (inquiry)	2,5	3	0	7,5
Module 4. Surface Integrals				
Current control			0	10
Classroom work	1	10	0	10
Periodical control			0	7,5
Theoretical survey (inquiry)	2,5	3	0	7,5
Bonus points				
1. Student academic competition or essay contest			0	5
2. Volunteering assistance in administrating of student academic competition or essay contest			0	5
Attendance (points are deducted from the total amount of points scored)				
1. Attending lectures			0	-6
2. Attending practical classroom work (seminar, laboratory classes)			0	-10
Final control				
Exam			0	30
Total points			45	100

Rating – plan of the discipline

Analysis II

(the name of the discipline according to the working curriculum)

Direction 01.03.02 Applied Mathematics and Informatics

Grade 2, semester 4

Rating-plan No. 2 (exam)

Types of educational activities of students	Points for a specific task	Number of tasks per semester	Points	
			minimum	maximum
Module 1. Complex numbers. Functions of a complex variable.				
Analytic functions.				
Current control			0	10
Classroom work	1	10	0	10
Periodical control			0	10
Theoretical survey (inquiry)	2,5	4	0	10
Module 2. The integral of a function of a complex variable. Cauchy integral formula.				
Current control			0	15
Classroom work	1	15	0	15
Periodical control			0	10
Theoretical survey (inquiry)	2,5	4	0	10
Module 3. Laurent's series. Residue.				
Current control			0	15
Classroom work	1	15	0	15
Periodical control			0	10
Theoretical survey (inquiry)	2,5	4	0	10
Bonus points				
1. Student academic competition or essay contest			0	5
2. Volunteering assistance in administrating of student academic competition or essay contest			0	5
Attendance (points are deducted from the total amount of points scored)				
1. Attending lectures			0	-6
2. Attending practical classroom work (seminar, laboratory classes)			0	-10
Final control				
Exam			0	30
Total points			45	100