RUSSIAN MINISTRY OF SCIENCE AND EDUCATION

FEDERAL STATE BUDGETARY EDUCATIONAL INSTITUTION
OF HIGHER EDUCATION
«BASHKIR STATE UNIVERSITY»

FACULTY OF MATHEMATICS AND INFORMATION TECHNOLOGIES

Approved: at the department meeting
Protocol # 5 from February 28, 2022
Head of the department
_______________________ Z. Yu. Fazullin

Coordinated with:
EMC chairman of the faculty/institute
_____________ A.M. Efimov

WORKING PROGRAM OF DISCIPLINE (MODULE)

Discipline Analysis III
(name of the discipline)

Obligatory part
(name of the part enclosing the discipline (obligatory, formed by participants of the educational activity, facultative))

bachelor (undergraduate) program

Course of training (speciality)
01.03.02 Applied mathematics and informatics
(code and name of the course of training (speciality))

Subdivision of the course of training (profile)
Applied programming and data analysis
(name of the profile of training)

Qualification (level of training)
bachelor
(name of the level of training)

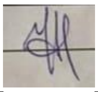
Designer (compiler): associate professor of the MA department, PhD	__________________ A.A. Makhota

For enrollment of: 2021

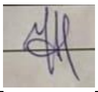
Ufa 2022

Designer: associate professor, PhD Alla Aleksandrovna Makhota

The working program of the discipline is approved at the meeting of the department of Mathematical Analysis,
protocol # 5 from February , « 28 » 2022.

Head of the department __________________ Z. Yu. Fazullin

The addenda and updates introduced into the working program of the discipline are approved at the meeting of the department of Higher algebra and geometry,
protocol # 11 from June , « 10 » 2022.

Head of the department __________________ Z. Yu. Fazullin

List of documents and proceedings

1. The list of planned learning outcomes of the discipline, correlated with the planned results of the educational program

Mastering the discipline must lead to forming of the following competence (GPC-1):

Category (group) of competencies (if there is exist a GPC)	Competence to be formed (with the code)	Code and name of the indicator of competence achievement	Learning outcomes for the discipline
Theoretical and practical basics of professional activity	*GPC-1. Able to apply the fundamental knowledge obtained in the mathematical and/or natural sciences and use them in professional activities*	*GPC-1.1. Possesses basic knowledge obtained in the field of mathematical analysis.*	*Know the basic concepts of disciplines, basic theorems and consequences, methods of solution and analysis of typical problems*
		GPC-1.2. Is able to use them in professional activities.	*Be able to use in practice the knowledge of disciplines, correctly formulate tasks and reasonably choose methods of their solutions*
		GPC-1.3. Has the skills to choose the methods of solving the problems of professional based on theoretical knowledge.	*Master the basic mathematical knowledge and its application to the solution of problems theoretical and applied character*

2. The discipline "Applied functional analysis" belongs to the basic part. The discipline is studied at the 3rd course in semesters 5-6. It’s a continuation of the Mathematical analysis and the Complex analysis.

The objectives of the discipline: the formation of mathematical culture of students; mastering the modern apparatus of analysis for further use in other areas of mathematics and disciplines of the natural sciences cycle.

3. The content of the working program (the volume of the discipline, types and types of classes, educational and methodological support of independent work of students)

The content of the working program is presented in Appendix № 1.

4. Evaluation funds for the discipline

4.1. List of competencies and indicators of achievement of competencies with the planned learning outcomes related to them in the discipline. Description of criteria and scales for evaluating the results of training in the discipline.

Code and definition of each competence.

GPC-1: Ability of applying the fundamental knowledge gained in the field of mathematical and (or) natural sciences and using them in professional activities.

Level of competence acquirement	**Planned learning outcomes (indicators of achievement of a predetermined level of acquired competencies)**	**Evaluation criteria for the results**			
		2 (" Non-satisfactory")	3 ("Satisfactory")	4 ("Good")	5 ("Excellent")
First level	To know: basic concepts of disciplines, basic theorems and their consequences, methods of solving and analyzing typical problems	Lack of knowledge	Incomplete ideas about the basic concepts of the discipline, the main theorems and their consequences, methods of solving and analyzing typical problems	Formed knowledge and skills, but containing separate gaps in the basic concepts of the discipline, the main theorems and their consequences, methods of solving and analyzing typical problems	Formed systematic knowledge and skills on the basic concepts of the discipline , the main theorems and their consequences, methods of solving and analyzing typical problems

Second level	Be able to: use the knowledge of the disciplines in practice, correctly formulate tasks and reasonably choose methods of their solution	Lack of skills	Partial skills of using the knowledge of the disciplines in practice, correctly formulate tasks and reasonably choose methods of their solution	Generally formed skills, but containing some gaps in the ability of using knowledge of the disciplines in practice, correctly formulate tasks and reasonably choose methods of their solution	Well-formed ability using the knowledge of the disciplines in practice, correctly formulate specific tasks and reasonably choose methods of their solution
Third level	Be in possession of: basic mathematical knowledge and its application for solving theoretical and applied problems	Lack of formed skills	Generally successful, but non-systematic application of basic mathematical knowledge for solving theoretical and applied problems	Generally successful, but containing some gaps in the application of basic mathematical knowledge for solving theoretical and applied problems	Successful acquisition of basic mathematic al knowledge and its application for solving theoretical and applied problems

Level of competence acquirement	Planned learning outcomes (indicators of achievement of a predetermined level of acquired competencies)	Evaluation criteria for the results (pass/fail rating)	
		Grade of "non pass" (**“failed”**)	**Grade of "pass"** (**“passed”**)
First level	To know: basic concepts of the disciplines, basic theorems and their consequences, methods of solving and analyzing typical problems	Incomplete ideas about the basic concepts of the discipline, the main theorems and their consequences, methods of solving and analyzing typical problems	Well-formed systematic ideas about the main concepts of the discipline, the main theorems and their consequences, methods of solving and analyzing typical problems
Second level	Be able to: use the knowledge of the disciplines in practice, correctly formulate tasks and reasonably choose methods of their solution	Partial skills of using the knowledge of the disciplines in practice, correctly formulate tasks and reasonably choose methods of their solution	Well-formed ability of using the knowledge of the disciplines in practice, correctly formulate tasks and reasonably choose methods of their solution
Third level	Be in possession of: basic mathematical knowledge and its application for solving theoretical and applied problems	Generally successful, but non systematic application of basic mathematical knowledge for solving theoretical and applied problems	Successful acquirement of basic mathematical knowledge and its application for solving theoretical and applied problems

Evaluation criteria are the points which are given by the teacher to each student for different types of activities (evaluation tools) at the end of the study of each module (section of a discipline) that are listed in the rating plan of the discipline (for the exam: current control –40 points maximum; periodical control – 30 points maximum, bonus points – 10 points maximum. For the grade of “pass”: current control - 50 points maximum; periodical control – 50 points maximum, bonus points – 10 points maximum).

Grading scale:

(for the exam:

45 to 59 points – "satisfactory";

60 to 79 points – "good";

80 points – "excellent".

for the grade of “pass”:

“passed” - **from 60 to 110** rating points (including 10 bonus points),

“failed” - **from 0 to 59** rating points).

4.2. Standard control tasks or other materials necessary for evaluating the results of training in the discipline, correlated with the indicators of competence achievement established in the educational program. Methodological materials defining the procedures for evaluating the results of training in the discipline

Indicator of the achieved competence	Results of training in the discipline	Evaluation tools
GPC-1. 1. Possesses basic knowledge gained in the field of mathematical analysis	To know: the basic concepts of the discipline, the main theorems and their consequences, methods of solving and analyzing typical problems	Theoretical survey, exam
GPC-1. 2. Knows how to use them in professional activities	Be able to: use the knowledge of the discipline in practice, correctly formulate tasks and reasonably choose methods of their solution	Laboratory work, test work, pass/fail rating
GPC-1.3. Possesses the skills to choose methods of solving problems in professional activity based on theoretical knowledge.	To possess: basic mathematical knowledge and its application for solving theoretical and applied problems	Laboratory work, test work, pass/fail rating

Examination tickets

Exam and credit is an assessment tool for all stages of mastering the competencies.

Questions for the exam (6 semester):

1. Metric spaces (MS): definition, examples ($\mathbb{R}^n, \mathbb{C}[a,b], l^2$), continuous mappings, homeomorphism, isometry.
2. Closure of sets, closed sets, convergence in MS.
3. Dense and everywhere dense sets in MS. Separable MS. Examples of separable and non-separable(!) MS.
4. Completeness of MS. Proof of completeness of space l^2, incompleteness of $\mathbb{C}[a,b]$ with integral metrics (proof).
5. Completeness. Theorem on completion (without proof). The principle of imbedded balls.
6. The principle of contraction mappings in MS. Examples of applications.
7. Normed spaces, examples with verification of axioms of the norm.
8. Euclidean spaces (definition, examples). Cauchy-Bunyakovsky inequality in Euclidean spaces. Norm in ES (checking the norm axioms).
9. Euclidean spaces (definition, examples). Continuity of linear operations and scalar product in ES with respect to an introduced norm
10. The angle between vectors in ES. Orthogonality. Orthogonal, orthonormalized systems, basis. Linear dependence and independence of systems of elements. Lemma on linear independence of an orthogonal system of elements which does not contain a zero element in an ES.

11. Angle between vectors in ES. Orthogonality. Orthogonal, orthonormalized system, basis. An orthogonal system in a separable ES is no more than countable (lemma with proof).
12. Angle between vectors in an ES. Orthogonality. Orthogonal, orthonormalized system, basis. Orthogonalization theorem.
13. Angle between vectors in ES. Orthogonality. Orthogonal, orthonormalized system, basis. Existence of an orthonormalized basis in a separable ES.
14. Bessel's inequality and Parseval's equality. Closedness and completeness equivalence for an orthonormalized system in a separable ES.
15. Euclidean spaces. The Riess-Fisher theorem
16. Hilbert space. Example. Orthogonal addition to a set in GS.
17. Hilbert space. Theorem on decomposition of GS into a direct sum of orthogonal subspaces
18. Hilbert spaces. Isomorphism theorem for GS
19. Linear, continuous, bounded functionals in a normal space.
20. Theorem on the general form of a linear continuous functional in GS.
21. Self-conjugate operators in GS. The Hilbert-Schmidt theorem (without proof).

Structure of the examination ticket:

1. Theoretical question.
2. Theoretical question

Grading criteria (in points):

- **25-30 points** if student demonstrates the knowledge of 80% or more of the required educational material in the discipline.

- ***17-24*** **points** if student demonstrates the knowledge from 60% to 79% of the required educational material in the discipline.

- 10-16 points if student demonstrates the knowledge from 45% to 59% of the required educational material in the discipline.

- 1-10 points if student demonstrates the knowledge less than 45% of the required educational material in the discipline.

An example of examination tickets is given in Appendix 2.

Sample questions for verbal examination

Verbal questioning is conducted in class on the list of examination questions, the answer is evaluated on a scale of "passed" - 1 point, "not passed" - 0 points.

Test

In the test - a score of 3 points for each correct answer, 0 points - incorrect answer. The maximum possible score is 36 points.

1) Which of the following sets $A \subset E$, where $E = [0;1] \times [0;1]$, is elementary set?

A) $A = \{(x; y) \in [2/5; 2/3] \times [0; 1/2], \quad x < y\}$.

B) $A = \left((1/4; 1/2] \times [0;1)\right) \bigcup \left([0;1] \times (1/3; 5/7)\right)$.

C) $A = \{(x; y) \in E : |x| < 1/2, 0 \le y \le x\}$.

D) $A = \{(x; y) \in E : 0 < x \le 1/4, y = 2x\}$.

E) All sets A)-D) are not elementary sets.

2) The measure of an elementary set always

A) depends on its area;
B) is not equal to its area;
C) is less than the sum of the areas of its constituent rectangles;
D) is equal to twice its area;
E) Answers A)-D) are not true.

3) The lower measure can be calculated

A) only for elementary sets;
B) not only for rectangles;
C) not for any set $A \subset E$;
D) only for countable unions of elementary sets;
E) statements A)-D) are not true.

4) The set $A \subset E$ is measurable if and only if

A) it is nothing more than a countable union of elementary sets;
B) $\forall \varepsilon > 0$ $\exists$ the elementary set B_ε: $\mu^*(A \Delta B_\varepsilon) > \varepsilon$;

C) $\forall \varepsilon > 0$ $\exists$ the elementary set B_ε: $\mu^*(A \setminus B_\varepsilon) < \varepsilon$;

D) $\forall \varepsilon > 0$: $\exists$ the elementary set $B_\varepsilon : \mu^*(A \Delta B_\varepsilon) < \varepsilon$;

E) statements A)-D) are not true.

5) The sets $A, B \subset E$ are measurable, $\mu(A) = 1/4; \quad \mu(B) = 1/2, \quad \mu(A \bigcap B) = 1/8.$ Find $\mu(A \Delta B.)$

A) 1/2; B) 1/4; C) 1; D) 3/8; E) 7/8.

6) Let $f: A \to \mathbb{R}$, $A \subset E$ -- is a measurable Lebesgue set. Choose the correct statement.

A) If f is measurable, the set of its values is infinite.

B) If f is measurable, the set of its values is no more than countable.

C) If the set of values of the function f is countable, then it is simple.

D) If f -- is a simple function, the set of its values is finite.

E) All of the statements are not true.

7) Let $f:[-1;1] \to \mathbb{R}$, $f(x) = \{x^2\}$. The set $A_{0.25} = f^{-1}((-\infty; 0.25))$ equals

A) $[0; 0.25)$; B) $[0;1/\sqrt[3]{2})$; C) $(-1/2;1/2)$; D) $[0;1/2)$; E) there is no correct answer.

8) Functions f,g are defined and measurable on the [0;1]. Functions f,g are equal almost everywhere on the [0;1], if

I. $f(x) = g(x), \quad \forall x \in (0;1]$;

II. $f(x) = g(x) \quad \forall x \in (0;1) \setminus \mathbb{Q}$;

III. $f(x) = g(x) \quad \forall x \in (0;1) \bigcap \mathbb{Q}$;

IV. $f(x) = g(x) \quad \forall x \in [0;1] \bigcap \mathbb{Q}$.

Which of the following statements is true?
A) II., III;
B) I., III.;
C) I., II., IV;
D) I., IV.;
E) I., II.

9) Let $f:[-1;1] \to \mathbb{R}$ defined as

$$f(x) = \begin{cases} -1, \ x \in \left([-1; \frac{1}{2}] \bigcap \mathbb{Q} \right) \bigcup \left((\frac{1}{2}; 1] \setminus \mathbb{Q} \right), \\ 0, \ x \in \left([-1; \frac{1}{2}] \setminus \mathbb{Q} \right) \bigcup \left((\frac{1}{2}; 1] \bigcap \mathbb{Q} \right). \end{cases}$$

Calculate $\int\limits_{[0;1]} f \, d\mu$.

A) $-\frac{1}{4}$; B) 0; C) $\frac{1}{2}$; D) $\frac{1}{4}$; E) $-\frac{1}{2}$.

10) Let $f:[0;2]\to\mathbb{R}$ defined as $f(x)=[x+1]$.

Calculate $\int\limits_{[0;1]} f\,d\mu$.

A) 1; B) 3; C) $\frac{3}{2}$; D) 2; E) 0.

11) Let $f:[-\pi;\pi]\to\mathbb{R}$ defined as $f(x)=[\sin(x/2)]$.

Calculate $\int\limits_{[-\pi;\pi]} f\,d\mu$.

A) π; B) $-\pi$; C) 1; D) 0; E) -1.

12) Let $f:A\to\mathbb{R}$ -- is measurable function, defined on a measurable set $A\subset E$.

Which of the following statements is true?

A) If $\exists\int\limits_A |f(x)|\,d\mu$, then $\exists\int\limits_A f(x)\,d\mu$.

B) If $\exists\int\limits_A f(x)\,d\mu$, then $\exists\int\limits_A |f(x)|\,d\mu$.

C) If $\int\limits_A |f(x)|\,d\mu=0$, then $\int\limits_A f(x)\,d\mu=0$.

D) If $\int\limits_A f(x)\,d\mu=0$, then $\int\limits_A |f(x)|\,d\mu=0$.

E) All statements are true.

Assignments for test work

For each task of the test work are given from 0 to 3 points. At the same time 0 points - no solution or incorrect solution, the maximum point for the task - a complete correct solution. An intermediate grade, in steps of 1 (1,2,3) or in steps of 0,5 (0,5, 1, 1,5, 2) is assigned for the presence of an incomplete solution, and depending on the flaws and inaccuracies, individually in each case.

Examples of a variant of the test work:

Problem 1: Let $X = C[0,1]$ and $\|x\| = \int_0^1 |x(t)|\, dt$,

- **a)** check the axioms of the norm;
- **b)** find $\|x\|$, if $x = \ln(t+1)$.

Problem 2: a) orthogonalize and then normalize the system of elements $x_1(t) = t; x_2(t) = \cos \pi t$ in Hilbert space $x = L^2(0,2)$ over $\mathbb{R}$;

b) in Hilbert space $x = L^2(0,2)$ over $\mathbb{R}$ find the orthogonal addition $M^\perp$, if $M = \{x(t): x(1+t) = x(1-t), \forall t \in (0,1)\}$.

Calculating and graphic work

For which λ we apply the principle of contraction mappings in spaces $L^2(a,b)$ and $C[a,b]$ to the Fredholm equation of the 2nd kind,

$$x(t) = \lambda \int_a^b K(t,s)x(s)\, ds + y(t).$$

Find the exact solution of the equation. To find the approximate solution make a program. In the program provide for:

a) tabulating the exact solution in steps $h = \frac{b-a}{2n}$;

b) calculation of the number L of iterations providing the accuracy of 0, 01 in the metric $C[a,b]$, when the first approximation is taken as $x_1(t) \equiv y(t)$. Note that L is determined from the inequality

$$\frac{\theta^{L-1}}{1-\theta}\rho(x_1, x_2) < 0{,}01$$

where θ is the compression coefficient;

c) comparison of the exact solution with the approximated one, i.e., computation

$$R = \max_{0 \le k \le 2n} |x(t_k) - \bar{x}(t_k)|,$$

where $x(t_k)$ – exact solution, $\bar{x}(t_k)$ are the values of the approximate solution.

Table 1

k	0	1	2	$\dots$	$2n$
$x(t_k)$					
$\overline{x}(t_k)$					

Table 2

№№	$K(t,s)$	$y(t)$	a	b	λ	n
1.	t^2s	$\cos 3t$	0	1	1	8
2.	t^2s	$\cos 3t$	0	1	1	10
3.	ts	e^t	-1	1	0,5	10
4.	ts	e^t	0,5	1,5	0,5	10
5.	t^2s^2	$\frac{\sin t}{t}$	0,3	1,2	0,6	9
6.	e^{t-s}	1	0,1	0,9	0,5	8
7.	e^{t-s}	t	0,1	0,8	0,5	7
8.	$\cos\pi(t-s)$	1	0	1	0,5	10
9.	$\cos t\ \sin s$	1	0	2	0,25	10
10.	$\sin t\ \cos s$	1	-0,5	0,5	0,25	5
11.	$e^{2t}\cos s$	$\sin t$	0	0,5	0,6	5
12.	$e^{2t}\sin s$	$\cos t$	0	0,5	0,4	10
13.	$e^t\cos s$	$2\sin t$	0	0,9	0,5	8
14.	$e^t\sin s$	$2\cos t$	0	0,8	0,5	8
15.	$t\sin s$	e^t	0,2	0,8	0,4	6
16.	$t\cos s$	e^{-t}	0,1	0,9	0,3	8
17.	t^2e^s	t	-2	-1	2	6

4.3. Rating – plan of the discipline

Rating-plan of the discipline is given in Appendix 3.

5. Educational, methodic and informational support of the discipline.

5.1. List of references to primary and complementary educational literature necessary for acquiring the discipline.

Primary literature

1. Элементы теории функций и функционального анализа [Электронный ресурс]: учебное пособие / А.Н. Колмогоров, С.В. Фомин. — Электрон. дан. — Москва: Физматлит, 2009. — 572 с. https://e.lanbook.com/book/2206.
2. Функциональный анализ [Электронный ресурс]: лабораторные работы для студентов 3 курса факультета математики и информационных технологий / Башкирский государственный университет; сост. Р.А. Башмаков; Н.Н. Аиткужина; А.А. Махота. — Уфа: РИЦ БашГУ, 2017. — Электрон. версия печ. публикации. — Доступ возможен через Электронную библиотеку БашГУ. — https://elib.bashedu.ru/d
3. Люстерник Л.А. Краткий курс функционального анализа [Электронный ресурс]: учебное пособие / Л.А. Люстерник, В.И. Соболев - СПб: Лань, 2009 - 270. http://e.lanbook.com

Auxiliary literature:

4. Р. А. Башмаков, А. А. Махота, Р. С. Юлмухаметов Мера и интеграл [Электронный ресурс]: курс лекций /; БашГУ. — Уфа: Изд-е БашГУ, 2012. — Электрон. версия печ. публикации. — Доступ возможен через Электронную библиотеку БашГУ. https://elib.bashedu.ru/dl/corp/BashmakovMera i IntegralKursLekcii.2012.pdf
5. Гуревич А. П., Корнев В. В., Хромов А. П. — Сборник задач по функциональному анализу. СПб: Лань, 2012
6. T.B. Ward Functional analysis lecture notes. School of Mathematics, University of East Anglia, Norwich NR4 7TJ, U.K. http://dsp-book.narod.ru/Fa.pdf

5.2. List of the Internet resources and software necessary for acquiring the discipline, including professional data bases and reference systems.

1. Library of Bashkir State University http://lib.bashedu.ru
2. BashSU Electronic Library System https://elib.bashedu.ru
3. Universitary WebWork server: `http://webwork-okko.bashedu.ru/webwork2/`.

6. Hardware equipment, materials and rooms necessary for implementing the educational process in the discipline.

Names of specialized rooms, rooms and laboratories	*Activity form*	*Name of the equipment/software*
1	*2*	*3*
Rooms 501,517, 528 or any other room according to the current time table	*Lectures*	The board for writing, projector and screen
Rooms 517, 503 or any other room according to the current time table	*Laboratory/practical classes*	The board for writing, projector and screen
Library, reading halls	*Individual work*	Internet. The universitary WebWork server

Appendix 1

FEDERAL STATE BUDGETARY EDUCATIONAL INSTITUTION
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CONTENT OF THE WORKING PROGRAM

of the discipline Analysis III for semester 5

Activity	**Duration**
Total duration of the discipline (CUD / hours)	4/144
Academic hours for the work with instructor	
lectures	36
laboratory	36
practical classes	-
other (consultation in group or individually and other forms of learning activities assuming collaboration of learners with instructor)	0,2
Academic hours for individual work of learners	71,8
Academic hours for preparing to exam/credit test/differentiated credit test (Grading)	-
laboratory	

Final grading:

the grade of "pass" in semester 5

Appendix 1

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CONTENT OF THE WORKING PROGRAM

of the discipline Analysis III for semester 6

Activity	Duration
Total duration of the discipline (CUD / hours)	5/180
Academic hours for the work with instructor	
lectures	32
laboratory	32
practical classes	-
other (consultation in group or individually and other forms of learning activities assuming collaboration of learners with instructor)	1,7
Academic hours for individual work of learners	79,5
Academic hours for preparing to exam/credit test/differentiated credit test (Grading)	34,8
laboratory	

Final grading:

exam in semester 6

Semester 5								
Item no.	Topic and its content	Learning forms: lectures, seminars, laboratory, and individual work with duration (in academic hours)				Primary and auxiliary literature (numbers in the reference list)	Task for individual work of learners	Forms of current grading (clolloquia, quizes, computer tests etc)
		LEC	SEM	LAB	IND			
1	2	3	4	5	6	7	8	9
1.	Elements of set theory (repetition)	2		2	4	1,2,5	Tasks are given by the teacher	Theoretical survey Laboratory work
2.	Lebesgue measure in $\mathbb{R}^2$: elementary sets and their properties, upper and lower measures of a set, measurable sets, measurability criterion, (countable) additivity and continuity of Lebesgue measure.	6		4	10	1,2,5	Tasks are given by the teacher	Theoretical survey Laboratory work
3.	Measurable functions and their properties. Different types of convergence of a sequence of measurable functions, the relationship between them.	4		4	9	1,2,5	Tasks are given by the teacher	Laboratory work, verbal questioning
4.	The Lebesgue integral of a simple function and its properties, the Lebesgue integral of an arbitrary measurable function.	4		4	6	1,2,5	Tasks are given by the teacher	Theoretical survey Laboratory work

5.	Limit transition under the sign of Lebesgue integral, relation between Riemann and Lebesgue integrals.	4		4	8,8	1,2,5	Tasks are given by the teacher	Laboratory work, verbal questioning
6.	Metric spaces: definition, examples of basic spaces and metrics in them. Convergence of a sequence. Closure, closed and open sets. Continuous mappings of m.p.	4		6	10	1,3,4,5	Tasks are given by the teacher	Laboratory work, verbal questioning
7.	Completeness and supplement. The principle of imbedded balls. Separability.	4		4	8	1,3,4,5	Tasks are given by the teacher	Laboratory work, verbal questioning
8.	The principle of contraction mappings. The method of successive approximations.	4		4	8	1,3,4,5	Tasks are given by the teacher	Laboratory work, verbal questioning Calculating and graphic work
9.	Normalized spaces: definition, examples, linear continuous functionals. The Hahn-Banach theorem in n.p.(without proof)	4		4	8	1,3,4,5	Tasks are given by the teacher	Laboratory work, verbal questioning
	Total Hours	**36**	**-**	**36**	**71,8**			

Semester 6								
Item no.	Topic and its content	Learning forms: lectures, seminars, laboratory, and individual work with duration (in academic hours)				Primary and auxiliary literature (numbers in the reference list)	Task for practical/individual work of learners	Forms of current grading (clolloquia, quizes, computer tests etc)
		LEC	SEM	LAB	IND			
1	2	3	4	5	6	7	8	9
1.	Euclidean spaces (definition, examples). The Cauchy-Bunyakovsky inequality in Euclidean space. Norm in ES. Continuity of linear operations and scalar product in ES with respect to the introduced norm. The angle between vectors in ES. Orthogonality. Orthogonal, orthonormalized system, basis. Theorem on orthogonalization and its consequences.	7		6	18	1,3,5	Tasks are given by the teacher	Theoretical survey, Laboratory work Verbal questioning
2.	Bessel's inequality and Parseval's equality. Closedness and completeness equivalence for an orthonormalized system in a separable ES. The Riess-Fisher theorem.	9		8	20,5	1,3,5	Tasks are given by the teacher	Verbal questioning Laboratory work

3.	Hilbert space. The theorem on decomposition of GPs into a direct sum of orthogonal subspaces. Theorem on isomorphism of the GS. A general form of a linear continuous functional in GS.	8		10	21	1,3,5	Tasks are given by the teacher	Laboratory work Verbal questioning
4.	Conjugate operator. Self-conjugate operators in GS. The Hilbert-Schmidt theorem (without proof).	8		8	20	1,3,5	Tasks are given by the teacher	Theoretical survey, Laboratory work
	Total hours:	32	-	32	79,5			

Appendix 2

An example of examination tickets (semester 6)

FEDERAL STATE BUDGETARY EDUCATIONAL INSTITUTION
OF HIGHER EDUCATION
«BASHKIR STATE UNIVERSITY»
FACULTY OF MATHEMATICS AND INFORMATION TECHNOLOGIES
Department of Mathematical Analysis

EXAMINATION TICKET #1
on "Analysis III"

1. Completeness. Theorem on completion (without proof). The principle of imbedded balls.

2. Self-conjugate operators in GS. The Hilbert-Schmidt theorem (without proof).

Appendix 3

Rating – plan of the discipline

Analysis III

(the name of the discipline according to the working curriculum)

Direction 01.03.02 Applied Mathematics and Informatics
Grade 3, semester 5

Rating-plan No. 1 (*the grade of "pass"*)

Types of educational activities of students	Points for a specific task	Number of tasks per semester	Points	
			minimum	maximum
Module 1. Theory of measure.				
Current control			**0**	**24**
Classroom work	**1**	**24**	**0**	**24**
Periodical control			**0**	**36**
Test	**3**	**12**	**0**	**36**
Module 2. Metric space				
Current control			**0**	**25**
Classroom work	**1**	**25**	**0**	**25**
Periodical control			**0**	**15**
Control test	**2,5**	**2**	**0**	**5**
Control test	**2,5**	**4**	**0**	**10**
Bonus points				
1. Student academic competition or essay contest			**0**	**5**
2. Volunteering assistance in administrating of student academic competition or essay contest			**0**	**5**
Attendance (points are deducted from the total amount of points scored)				
1. Attending lectures			**0**	**-6**
2. Attending practical classroom work (seminar, laboratory classes)			**0**	**-10**
Final control				
Total points			**-16**	**110**

Rating – plan of the discipline

Analysis III

(the name of the discipline according to the working curriculum)

Direction 01.03.02 Applied Mathematics and Informatics
Grade 3, semester 6

Rating-plan No. 2 (exam)

Types of educational activities of students	Points for a specific task	Number of tasks per semester	Points	
			minimum	maximum
Module 1. **Normed and Hilbert spaces.**				
Current control			**0**	**30**
Classroom work	**1**	**30**	**0**	**30**
Periodical control			**0**	**18**
Theoretical survey (inquiry)	**3**	**6**	**0**	**18**
Module 2. Linear functionals and operators in Hilbert spaces				
Current control			**0**	**10**
Classroom work	**1**	**10**	**0**	**10**
Periodical control			**0**	**12**
Test work	**3**	**4**	**0**	**12**
Bonus points				
1. Student academic competition or essay contest			**0**	**5**
2. Volunteering assistance in administrating of student academic competition or essay contest			**0**	**5**
Attendance (points are deducted from the total amount of points scored)				
1. Attending lectures			**0**	**-6**
2. Attending practical classroom work (seminar, laboratory classes)			**0**	**-10**
Final control				
Exam			**0**	**30**
Total points			**-16**	**110**