RUSSIAN MINISTRY OF SCIENCE AND EDUCATION

FEDERAL STATE BUDGETARY EDUCATIONAL INSTITUTION
OF HIGHER EDUCATION
«BASHKIR STATE UNIVERSITY»

FACULTY OF MATHEMATICS AND INFORMATION TECHNOLOGIES

Approved: at the department meeting
Protocol # 8 from February 28, 2022
Head of the department
_________________ Khabibullin B.N.

Coordinated with:
EMC chairman of the faculty/institute
_________________ Efimov A.M.

WORKING PROGRAM OF DISCIPLINE (MODULE)

Discipline Linear algebra and applications

(name of the discipline)

Obligatory part

(name of the part enclosing the discipline (obligatory, formed by participants of the educational activity, facultative))

bachelor (undergraduate) program

Course of training (speciality)
01.03.02 Applied mathematics and informatics
(code and name of the course of training (speciality))

Subdivision of the course of training (profile)
Applied programming and data analysis
(name of the profile of training)

Qualification (level of training)
bachelor
(name of the level of training)

Designer (compiler): associate professor of the HAG department, PhD	_________________ Sharipov R.A.

For enrollment of: 2022

Ufa 2022

Designer: associate professor, PhD Sharipov Ruslan Abdulovich.

The working program of the discipline is approved at the meeting of the department of Higher algebra and geometry,
protocol # 8 from February , « 28 » 2022.

Head of the department ____________________ Khabibullin B. N.

List of documents and materials

1. **List of expected results of education in the discipline correlated with the indicators of reaching the competencies.**

As a result of acquiring the educational program the learner should reach the following results in the discipline.

Competency being formed (with code)	Code and name of the indicator of reaching the competency	Learning results in the discipline
GPC-1 – Is able to apply fundamental knowledge acquired in the field of mathematics and/or natural sciences and to use them in professional activities.	*GPC-1.1 Knowing concepts.*	**<u>Semester 1</u>** <u>Mast know</u>: the concept of a linear equation and a system of linear equations, Gauss's and Cramer's methods for solving them; the concept of a matrix and matrix operations; the concept of the determinant of a matrix and the concept of the inverse matrix; the concept of a vector and algebraic operations with vectors; properties of the algebraic operations with vectors; the concepts of collinearity and co planarity; bases on a line, on a plane, and in the space; the concepts of the scalar, vectorial, and mixed products and their properties, methods of calculating these products in a skew-angular basis and in an orthonormal basis; contraction formulas; various forms of the equations of straight lines and planes; the canonic equations of ellipses, hyperbolas, and parabolas, the equations for tangent lines to them, location of their foci and directrixes, formulas for their eccentricity; the canonic equations for quadrics in the space. **<u>Semester 2</u>** <u>Mast know:</u> the concept of a linear vector space and its subspaces; the concept of bases and their changes; the conceit of sets and mappings among them; the concept of linear mappings and their matrices; the concept of linear operators and their matrices; the concept of eigenvalues and eigenvectors of a linear operator, the concepts of invariant spaces, eigenstates and root spaces of a linear operator; the concept of a Jordan form of the matrix of a linear operator; the concept of covectors, the concepts of bilinear and quadratic forms and their signatures; the concept of a multidimensional Euclidean space.
	GPC-1.2. Ability to operate with concepts..	**<u>Semester 1</u>** *<u>Must be able</u>: to add and multiply matrices*, to bring matrices to a Gaussian echelon form; to calculate the determinants of matrices and their inverse matrices; to add vectors and multiply them by numbers; to expand vectors in bases; to calculate the scalar, vectorial and mixed products through the coordinates of vectors in various bases; to recognize various forms of the equations or straight lines and planes and to transform any form of these

		equations to any other forms; to bring the equations of quadrics to their canonic forms; to recognize quadric surfaces s in the space by their canonic equations. **Semester 2** Must be able: to distinguish linear vector spaces and their subspaces from other sets; to perform changes of bases; to calculate sums and intersections of subspaces; to build mappings of sets; to calculate matrices of linear mappings and linear operators; to determine eigenvalues and eigenvectors of linear operators; bring the matrices of linear operators to their Jordan forms; co calculate scalar products of vectors and covectors; to determine signatures of quadratic forms; to calculate the lengths of vectors and the angles between them in multidimensional Euclidean spaces.
	GP-1.3. Ability to solve problems.	**Semesters 1 and 2** *Must have: the ability to combine theoretical knowledge with practical skills in solving educational training problems.*

2. Goal and role of the discipline in the structure of educational program.

The discipline «Linear algebra and applications» belongs to the obligatory part.
The discipline is studied in the first year, semesters 1 and 2.
The goal of learning the discipline: to acquire the mathematical apparatus used in specialization disciplines.
For learning the discipline the background of competencies formed in the previous level of education and verified during the enrollment at the university is required.

3. Content of the working program (duration of the discipline, sorts and forms of classes, educational and methodical support for the individual work of learners).

Content of the working program is given in Appendix 1.

4. Fund of grading materials in the discipline.

4.1. List of competencies and indicators of reaching competencies with expected learning results in the discipline correlated with them. Description of criteria and scales for grading the learning results in the discipline.

GPC-1 – Is able to apply fundamental knowledge acquired in the field of mathematics and/or natural sciences and to use them in professional activities.

Code and name of the indicator of reaching the competency	**Learning results in the discipline**	Grading criteria for learning results			
		«Unsatisfactory»	«Satisfactory»	«Good»	«Excellent»
GPC-1.1 Knowing concepts.	**<u>Semester 1</u>** <u>Mast know</u>: the concept of a linear equation and a system of linear equations, Gauss's and Cramer's methods for solving them; the concept of a matrix and matrix operations; the concept of the determinant of a matrix and the concept of the inverse matrix; the concept of a vector and algebraic operations with vectors; properties of the algebraic operations with vectors; the concepts of collinearity and coplanarity; bases on a line, on a plane, and in the space; the concepts of the scalar, vectorial, and mixed products and their properties, methods of calculating these products in a skew-angular basis and in an orthonormal basis; contraction formulas; various forms of the equations of straight lines and planes; the canonic equations of ellipses, hyperbolas, and parabolas, the equations for tangent lines to them, location of their foci and directrices, formulas for their excentricitet; the canonic equations for quadrics in the space. **<u>Semester 2</u>** <u>Mast know:</u> the concept of a linear vector space and its subspaces; the concept of bases and their changes; the conceit of sets and mappings among them; the concept of linear mappings and their matrices; the concept of linear operators and their matrices; the concept of eigenvalues and eigenvectors of a linear operator, the concepts of invariant spaces, eigenspaces and root spaces of a linear operator; the concept of a Jordan form of the matrix of a linear operator; the concept of covectors, the concepts of bilinear and quadratic forms and	Practically does not know	Has substantial gaps in the knowledge	Knows almost all	Knows all

	their signatures; the concept of a multidimensional Euclidean space.				
GPC-1.2. Ability to operate with concepts.	**<u>Semester 1</u>** *<u>Must be able</u>: to add and multiply matrices*, to bring matrices to a Gaussian echelon form; to calculate the determinants of matrices and their inverse matrices; to add vectors and multiply them by numbers; to expand vectors in bases; to calculate the scalar, vectorial and mixed products through the coordinates of vectors in various bases; to recognize various forms of the equations or straight lines and planes and to transform any form of these equations to any other forms; to bring the equations of quadrics to their canonic forms; to recognize quadric surfaces s in the space by their canonic equations. **<u>Semester 2</u>** <u>Must be able:</u> to distinguish linear vector spaces and their subspaces from other sets; to perform changes of bases; to calculate sums and intersections of subspaces; to build mappings of sets; to calculate matrices of linear mappings and linear operators; to determine eigenvalues and eigenvectors of linear operators; bring the matrices of linear operators to their Jordan forms; co calculate scalar products of vectors and covectors; to determine signatures of quadratic forms; to calculate the lengths of vectors and the angles between them in multidimensional Euclidean spaces.	Practically is unable	Is unable in most	Is able to do almost all	Is able to do all
GPC-1.3. Ability to solve problems.	**<u>Semesters 1 and 2</u>** <u>Must have:</u> the ability to combine theoretical knowledge with practical skills in solving educational training problems.	Practically does not have	Does not have in most	Has in essential	Has

GPC-1 – Is able to apply fundamental knowledge acquired in the field of mathematics and/or natural sciences and to use them in professional activities.

The form of the final grading in the discipline in both semesters 1 and 2 is an exam.

Criteria for grading are scores which are set by the instructor for various activities (grading tasks) in and upon learning modules listed in the Rating plan of the discipline.

from 45 up to 59 scores – «satisfactory»;
from 60 up to 79 scores – «good»;
from 80 scores up – «excellent».

4.2. Typical grading tasks or other materials required for grading the learning results in discipline correlated with the indicators of reaching the competencies which are set in the educational program. Methodical materials determining the grading procedures for learning results in the discipline.

Code and name of the indicator of reaching the competency	Learning results	Grading tasks
GPC-1.1 Knowing concepts.	**Semester 1** Mast know: the concept of a linear equation and a system of linear equations, Gauss's and Cramer's methods for solving them; the concept of a matrix and matrix operations; the concept of the determinant of a matrix and the concept of the inverse matrix; the concept of a vector and algebraic operations with vectors; properties of the algebraic operations with vectors; the concepts of collinearity and coplanarity; bases on a line, on a plane, and in the space; the concepts of the scalar, vectorial, and mixed products and their properties, methods of calculating these products in a skew-angular basis and in an orthonormal basis; contraction formulas; various forms of the equations of straight lines and planes; the canonic equations of ellipses, hyperbolas, and parabolas, the equations for tangent lines to them, location of their foci and directrices, formulas for their eccentricity; the canonic equations for quadrics in the space. **Semester 2** Mast know: the concept of a linear vector space and its subspaces; the concept of bases and their changes; the concept of sets and mappings among them; the concept of linear mappings and their matrices; the concept of linear operators and their matrices; the concept of eigenvalues and eigenvectors of a linear operator, the concepts of invariant spaces, eigenspaces	Problems for the midterm grading, exam topics and exam tickets, work at the board and in class room.

	and root spaces of a linear operator; the concept of a Jordan form of the matrix of a linear operator; the concept of covectors, the concepts of bilinear and quadratic forms and their signatures; the concept of a multidimensional Euclidean space.	
GPC-1.2. Ability to operate with concepts.	**<u>Semester 1</u>** *<u>Must be able</u>: to add and multiply matrices*, to bring matrices to a Gaussian echelon form; to calculate the determinants of matrices and their inverse matrices; to add vectors and multiply them by numbers; to expand vectors in bases; to calculate the scalar, vectorial and mixed products through the coordinates of vectors in various bases; to recognize various forms of the equations or straight lines and planes and to transform any form of these equations to any other forms; to bring the equations of quadric to their canonic forms; to recognize quadric surfaces s in the space by their canonic equations. **<u>Semester 2</u>** <u>Must be able:</u> to distinguish linear vector spaces and their subspaces from other sets; to perform changes of bases; to calculate sums and intersections of subspaces; to build mappings of sets; to calculate matrices of linear mappings and linear operators; to determine eigenvalues and eigenvectors of linear operators; bring the matrices of linear operators to their Jordan forms; co calculate scalar products of vectors and covectors; to determine signatures of quadratic forms; to calculate the lengths of vectors and the angles between them in multidimensional Euclidean spaces.	Problems for the midterm grading, exam topics and exam tickets, work at the board and in class room.
GP-1.3. Ability to solve problems.	**<u>Semesters 1 and 2</u>** <u>Must have:</u> the ability to combine theoretical knowledge with practical skills in solving educational training problems.	Problems for the midterm grading, exam topics and exam tickets, work at the board and in class room.

Topics for the exam in Semester 1.

1. Systems of linear equations. Gauss's method for solving them.
2. Systems of linear equations. Cramer's method for solving them.

3. Determinants of square matrices of an arbitrary size. Properties of determinants.
4. Multiplication of matrices. The conceit of an inverse matrix and methods for calculating it.
5. The concept of a vector. Geometric and free vectors. Algebraic operations with vectors. Properties of algebraic operations with vectors.
6. Collinearity, co planarity, and linear dependence of vectors.
7. Bases on a line, on a plane, and in the space. Uniqueness of the expansion of a vector in a basis.
8. Change of a basis. Transition matrices. Transformation of the coordinates of vector under a change of a basis.
9. Scalar product of vectors and its properties. Calculation of the scalar product through the coordinates of vectors in a skew-angular basis. Gram matrix. An orthonormal basis.
10. Vector product of vectors and its properties. Calculation of the vector product through the coordinates of vectors in an orthonormal basis.
11. Mixed (triple) product of vectors and its properties. Geometric interpretation of the mixed product. Oriented volume of a basis.
12. Calculation of the mixed product through the coordinates of vectors in a skew-angular basis.
13. Contraction of formulas.
14. Formula of the double vectorial product.
15. Equations of a straight line on a plane.
16. Equations of a plane in the space.
17. Equations of a straight line in the space.
18. Geometric definition of an ellipse and its canonic equation. Numeric parameters and geometric properties of an ellipse.
19. Geometric definition of a hyperbola and its canonic equation. Numeric parameters and geometric properties of a hyperbola.
20. Geometric definition of a parabola and its canonic equation. Numeric parameters and geometric properties of a parabola.
21. Bringing equations of quadrics to their canonic forms.
22. Quadric surfaces, their sorts and canonic equations.

Topics for the exam in Semester 2.

1. Complex numbers.

2. Linear vector spaces.
3. Linear dependence and linear independence.
4. Generating systems and bases. Coordinates of vectors.
5. Transformations of the coordinates of vectors under a change of a basis.
6. Intersections and sums of subspaces.
7. Sets and mappings.
8. Linear mappings.
9. Matrix of a linear mapping.
10. Linear operators.
11. Matrix of a linear operator.
12. Invariant subspaces. Restriction of operators.
13. Eigenvalues and eigenvectors of a linear operator.
14. Root subspaces, chain bases and Jordan form of the matrix of a linear operator.
15. Linear functionals. Vectors and covectors. Conjugate space.
16. Bilinear and quadratic forms. Recovery formula.
17. Bringing of a quadratic form to its canonic form. Indices of inertia and signature.

Exam tickets

Each exam ticket consists of 2 topics, the first topic is taken randomly from the first half part of the topic list, the second topic is taken randomly from the second half part of the corresponding topic list. Exhaustive talk on each topic is graded by 15 scores.

Grading criteria (in scores):

- **25-30 scores** if student demonstrates the knowledge of 80% or more of the required educational material in the discipline.
- ***17-24 scores*** if student demonstrates the knowledge from 60% to 79% of the required educational material in the discipline.
- **10-16 scores** if student demonstrates the knowledge from 45% to 59% of the required educational material in the discipline.
- **1-10 scores** if student demonstrates the knowledge less than 45% of the required educational material in the discipline.

An example of examination tickets is given in Appendix 4.

Problems for the midterm grading.

The discipline is subdivided into two modules in each of the two semesters. Each module has its list of problems. Problems are delivered online through the universitary WebWork server

http://webwork-okko.bashedu.ru/webwork2/

or through the testing system in student's personal account

https://cabinet.bashedu.ru/

The midterm grading can yield 30 scores, 15 per each of two modules. Examples of problems for the midterm grading are given in Appendix 3.

Work at the board and in class room.

The work at the board consists in selective solving some problems similar to those students get online through the universitary WebWork server. Solution of these problems is accompanied with the discussion of the theory. For each module student gets at the board at least once. His knowledge of the theory is graded by 5 scores, solution of problems is graded by 10 scores, addenda from the class room is graded by 5 scores. Totally 20 scores per each of the two modules.

Laboratory.

Laboratory work is organized as solving selected problems from the first and second WebWork tasks with a written report on each of them. It is graded separately and is not included to the grading of the discipline for semester.

Calculation and graphing.

Calculation and graphing work is organized as solving selected problems from the first and second WebWork tasks with a written report on each of them. It is graded separately and is not included to the grading of the discipline for semester.

4.3. Rating-plan of the discipline.

Rating-plan of the discipline is given in Appendix 2.

5. Educational, methodic and informational support of the discipline.

5.1. List of references to primary and complementary educational literature necessary for acquiring the discipline.

Primary literature:

1. Sharipov R. A. Course of analytical geometry. Learning textbook. // EPC of BSU, Ufa, 2011, C. 225. ISBN 978-5-7477-2574-4 [Electronic resource] — Electronic version of printed material .— <URL: https://elib.bashedu.ru/dl/local/Sharipov_Course_of_analitica_geometry_up_2011.pdf/info>.
2. Gaidamak O. G., Silova E. V. Analitical geometry and linear algebra. Study materials. // EPC of BSU, Ufa, 2012, C. 96. [Electronic resource] — Electronic version of printed material .—<URL: https://elib.bashedu.ru/dl/read/GaidamakSilovaAnalit.Geometriy i LineinayAlgebraUPos.2012.pdf/info>.
3. Sharipov R. A. Course of linear algebra and multidimensional geometry. Study materials. // EPC of BSU, Ufa, 1996, C. 146. ISBN 978-5-7477-0099-5 [Electronic resource] — Electronic version of printed material .— <URL: https://elib.bashedu.ru/dl/local/Sharipov_Kurs linejnoj algebry_up_1996.pdf>.

Auxiliary literature:

4. Akhmetvalieva E. N., Akhtyamov A. M. Mathematics. Part. 1: Elements of lnear algebra and analytical geometry. // EPC of BSU, Ufa, 2010 — Electronic version of printed material. — <URL: https://elib.bashedu.ru/dl/read/AhmetvalievaAhtymovaMatematika1Uch.pos.2010.pdf>.
5. Aleksandrov, P. S. Course of analytical geometry and linear algebra [Electronic resource]: textbook for phys. and math spec. of HEI / P. S. Aleksandrov.— SPb: Lan, 2009 .— 512 c. : ил. — ISBN 978-5-8114-0908-2 .— <URL:http://e.lanbook.com/books/element.php?pl1_id=493>.
6. Kartak V. V., Zerkina A. V. Higher algebra: methodical directions. Semester 2 // EPC of BSU, Ufa, 2013. — Electronic version of printed material. — <URL: https://elib.bashedu.ru/dl/corp/KartakVysshayaAlgebraI.pdf/info >.
7. Sadrieva R. TT., Sidelnikova N. A. Linear algebra. Methodical directions to solving problems from quises. // EPC of BSU, Ufa, 2013 — Electronic version of printed material. — <URL: https://elib.bashedu.ru/dl/corp/Sadrieva,Sidelnicova_coct_Lineynaya_algebra_Met.uk_Ufa_2013.pdf/info>.
8. Gumerov I. S., Murtazina S. A. Practicum on algebra. Typical problems and exercises for seminars and for organizing ndividual study of students. // EPC of BSU, Ufa, 2013 — Electronic version of printed material. — <URL: https://elib.bashedu.ru/dl/read/Gumerov, Murtazina_Praktikun_po_algebre_Tipov.zadachi_dlya_proved.prakt.zanyatiy_2013.pdf/info>.
9. Aleksandrov N. D. Lectures on algebra (univariate polynomials). // Birsk: Birsk branch of BSU, 2015. — Electronic version of printed material. — <URL: https://elib.bashedu.ru/dl/read/Alexandrov_avt-sost_Lexii_po_algebre_up_Birsk_2015.pdf/info>.

5.2. List of the Internet resources and software necessary for acquiring the discipline, including professional data bases and reference systems.

10. Universitary WebWork server: http://webwork-okko.bashedu.ru/webwork2/.
11. Shaipov R. A. Electronic course «Linapril FMiIT» in the system of distant learning of BSU: <URL: https://sdo.bashedu.ru/course/view.php?id=1553 >.

6. Hardware equipment, materials and rooms necessary for implementing the educational process in the discipline.

Names of specialized rooms, rooms and laboratories	*Activity form*	*Name of the equipment/software*
1	***2***	***3***
Rooms 01, 02, 301 or any other room according to the current time table	*Lectures*	The board for writing
Rooms 322, 318, 216 or any other room according to the current time table	*Seminars*	The board for writing
Library, reading halls	*Individual work*	Internet. The universitary WebWork server

Appendix 1

FEDERAL STATE BUDGETARY EDUCATIONAL INSTITUTION
OF HIGHER EDUCATION
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CONTENT OF THE WORKING PROGRAM

of the discipline Linear algebra and applications for semesters 1 and 2

full-time
learning form

Activity	**Duration**
Total duration of the discipline (CUD / hours)	11/396
Academic hours for the work with instructor	190
lectures	86
seminars	70
laboratory	34
other (consultation in group or individually and other forms of learning activities assuming collaboration of learners with instructor)	3,6
Academic hours for individual work of learners	105,8
Academic hours for preparing to exam/credit test/differentiated credit test (Grading)	96,6

Final grading:
exam in semester 1
exam in semester 2

Item no.	Topic and its content	Learning forms: lectures, seminars, laboratory, and individual work with duration (in academic hours)				Primary and auxiliary literature (numbers in the reference list)	Task for individual work of learners	• Forms of current grading (clolloquia, quizes, computer tests etc)
		LEC	SEM	LAB	IND			
1	2	3	4	5	6	7	8	9
Semester 1, Module 1								
1.	Systems of linear equatons (SLE). Matrix presentation of an SLE. Primary matrix (coefficient matrix) and extented matrix (augmented matrix) of an SLE. Elementary transformations of SLSs and elementary transformations of their matrices. Gauss's method for solving SLEs. Bringing the matrix of a system of linear equations to a Gaussian stair-like (echelon) form. Dependent and independent variables. Consistency of an SLE, rank of a matrix and Kronecker-Capelli theorem (Rouché-Capelli theorem). Cramer's method for solving SLEs. The concept of the determinant of a square matrix. Determinants of square matrices 2×2 and 3×3. Calculation of a determinant by beans of expanding by a row. Minors and algebraic complements (cofactors). Properties of determinants of arbitrary sizes. Calculation of determinants by means of elementary transformations of rows and columns in a matrix. Determinants of diagonal and triangular matrices.	6	6	2	8	1-3,11	10, first lonline WebWork task, problems 1-8 from the first midterm grading task	Scores for the work at the board and in class room
2.	Algebraic operations with matrices. Addition of	6	6	2	8	1-3,11	10, first online	Scores for the work at

	matrices, multiplication of matrices by numbers, multiplication of matrices by matrices. Writing multiplication of matrices in terms of their entries. Zero matriz and unit matrix. The concept of inverse matrix for square matrices. The determinat of the product of two square matrices. Non-degeneracy and ivertibility of square matrices. Geometric vectors and the operation of parallel translation. The concept of a free vector. Algebraic operations with free vectors: addition of vectors (the triangle rule and the parallelogram rule) and multiplication of vectors by numbers. Zero vector and the vector opposite to a given vector. Properties of the algebraic operations with vectors (8 properties). The concept of a linear combination. Coeficients and the value of a linear combinaton. Triviality and vanishing of linear combinations. The concept of lnear dependence and independence.						WebWork task, problems 9-16 from the first midterm grading task	the board and in class room
3.	Linear dependence for systes of one, two, and three vectors. The concepts of collinearity and coplanarity, their relation to linear dependence. Bases on a straight line, on a plane, and in the space. Drawing used for expanding a vector n a basis. Theorem on linear dependence of four and more vectors in our observable three-dimensional space. Bases and coordinate presentation of vectors. Theorem on uniqueness of the expansion of a vaector in a given basis. Column form of writing coordinates of a vector and upper indices. Change of bases. Transition formulas and transition matrices. Matrices of direct and inverse transitons. Transformation of the coordinates of a	6	6	2	8	1-3,11	10, first lonline WebWork task, problems 17-24 from the first midterm grading task	Scores for the work at the board and in class room

	vector under a change of a basis. Einstein's convension on setting indices in sums.							
4	Scalar products of vectors. Properties of the scalar product (4 properties). Calculations of the scalar product through the coordinates of vectors in a skew-angular basis. Gram matrix for a skew-angular basis. The concept of an orthonrmal basis. Calculations of the scalar product through the coordinates of vectors in an orthonormal basis. Kronecker symbol. Orientation. Concept of right and left triples of non-coplanar vectors. Vector product of vectors (three conditions determining the vector product). Properties of the vector product (4 properties). Calculation of the vector product through the coordinates of vectors in a skew angular basis (structure constants of the vector product). Structure constants of the vector product in the cases of a right and a left orthonormal bases. Calculation of the vector product through the coordinates of vectors in a right orthonormal basis (the formula in the form of the determinant). Using the vector product for calculating the area of a parallelogram and a triangle.	6	6	2	8	1-3,11	10, first online WebWork task, problems 25-34 from the first midterm grading task	Scores for the work at the board and in class room
Semester 1, Module 2								
5	Mixed product of vectors (triple product). Properties of the mixed product (4 properties). Calculation of the mixed product of vectors through the coordinates of vectors in a skew-angular basis. Structure constants of the mixed product. Oriented volume of a basis and Levi-Civita symbol. The expression of the structure constants of the mixed product through the Levi-Civita symbol. Calculation of the mixed product	6	6	2	8	1-3,11	10, second online WebWork task, problems 1-5 from the first midterm grading task	Scores for the work at the board and in class room

	of vectors through their coordinates in a right orthonormal basis (the formulas in the form of the determinant). Usage of the mixed product in calculating the volume of a skew-angular parallelepiped, a skew prism and a piramid. Contraction formulas. Sucsesive derivation of the first, the second, the trhird, and the fourth formulas of contraction. The formula of the double vectorial product and Jacobi identity. Usage of the contraction formulas for deriving the formula for double vectorial product. Other examples of using contraction formulas (the product of two mixed products). The relation of structure constants of vectorial and mixed products. Inverse Gram matrix. Raising and lowering indices. The expression of the structure constants of the vector product through the Levi-Civita symbol and the inverse Gram matrix.							
6	Bases and coordinate systems. The concept of the radius-vector. Transformation of the coordnates of a point under a change of a coordinate system. Rotation of a coordinate system. The rotation angle and the rotation matrix. Defining curves and surfaces by equations in coordinates. Parametric and non-parametric equations. A straight line on a plane. Various forms of the equation of straight line on a plane: 1) vectorial-parametric equation; 2) coordinate-parametric equation; 3) normal vectorial equation; 4) general equation in coordinates; 5) canonical equation in coordinates; 6) the equation of a straight line going through two given points; 7) the equation in segments.	6	6	2	8	1-3,11	10, second online WebWork task, problems 6-10 from the first midterm grading task	Scores for the work at the board and in class room
7	A plane in the space. Various forms of the equation of a plane in the space: 1) vectorial-	6	6	2	8	1-3,11	10, second online WebWork task,	Scores for the work at the board and in class

	parametric equation; 2) coordinate-parametric equation; 3) normal vectorial equation; 4) general equation in coordinates; 5) canonical equation in coordinates; 6) the equation of a straight line going through two given points; 7) the equation in segments. A straight line in the space. Various forms of the equation of a straight line in the space: 1) vectorial-parametric equation; 2) coordinate-parametric equation; 3) vetorial equation; 4) canonical equation in coordinates; 5) the equation of a straight line going through two given points; 6) a straight line as the intersection of two planes.						problems 11-5 from the first midterm grading task	room
8	Ellipse. Geometric definition and canonical equation of an ellipse. Vertices, semi-axes, foci, excentricity, and directrices of an ellipse. The equation of a tangent line to an ellipse. The properties of directrices and the focal property of an ellipse. Hyperbola. Geometric definition and canonical equation of a hyperbola. Vertices, semi-axes, foci, excentricity, and directrices of a hyperbola. The equation of a tangent line to a hyperbola. The properties of directrices and the focal property of a hyperbola. Asymptotes of a hyperbola.	6	6	2	8	1-3,11	10, second online WebWork task, problems 16-20 from the first midterm grading task	Scores for the work at the board and in class room
9	Parabola. Geometric definition and canonical equation of a parabola. The vertex, the focus and the numeric parameter of a parabola. The equation of a tangent line to a parabola. Focal property of a hfrabola. Curves of the second order on a plane. Bringing the equation of curve of the second order to its canonical form. Classification of curves of the second order (9 types, regular and degenerate	6	6	2	7,3	1-3,11	10, second online WebWork task, problems 21-28 from the first midterm grading task	Scores for the work at the board and in class room

	cases). Surfaces of the second order. Classification of surfaces of the second order (17 types, regular and degenerate cases).							
	Total hours for Semester 1:	**54**	**54**	**18**	**71,3**			

Item no.	Topic and its content	Learning forms: lectures, seminars, laboratory, and individual wrk with duration (in academic hours)				Primary and auxiliary litrature (numbers in the reference list)	Task for individual work of learners	Forms of current grading (clolloquia, quizes, computer tests etc)
		LEC	SEM	LAB	IND			
1	2	3	4	5	6	7	8	9
Semester 2 Module 1								
1.	Matrix solution of the equation $i^2=-1$. Complex numbers. Real and imaginary parts of a complex number. Addition and subtraction of complex numbers. Multiplication and division of complex numbers. Modulus and argument of a complex number. Trigonometric form of a complex number. Exponentiation and finding roots or higher degrees of a complex number. Complex logarithm and complex exponential function. Linear vector space (LVS). Definition and examples. Axioms of an LVS and simplest consequences of them. The concept of a subspace. Linear dependence and independence for systems of vectors in an LVS. Properties of lnear	4	2	2	4	1-3,12	10, first online WebWork task, problems 1-8 from the first midterm grading	Scores for the work at the board and in class room

	dependence. Steinitz theorem.							
2.	Systems of vectors and their spans. Generating systems of vectors for subspaces. Minimality and linear independence for generating systems. Bases and the dimension of an LVS. Properties of the dimension. Theotem on complementing of a basis. Bases and a coordinate presentation of vectors, upper and loer indices. Change of a basis in an LVS. Transition matrices. The relation of the direct and the inverse transition matrices. Transformation of the coordinates of a vector under a change of a basis.	4	2	2	4	1-3,12	10, first online WebWork task, problems 9-16 from the first midterm grading task	Scores for the work at the board and in class room
3.	Mappings. The domain and the codomain of a mapping. Images and total preimages of separate elements and of subsets. The range (image) of a mapping. Surjectivity, injectivity and bijectivity of mappings. Composition of mappings. The identity mapping. The concept of the inverse mapping. Restriction and extention of a mapping. Linear mappings. Kernel and image o a linear mapping. Kriteria of injectivity and surjectivity of linear mappings in terms of its kernel and its image. Linearity of the mapping inverse to a bijective linear mapping. Theorem on linear independence of preimages of linearly independent vectors. Isomorphism of linear vector spaces. Theorem on the coincidence of the dimensions of isomorphic spaces. Bases and the isomorphism of general LVSs to the spaces $\mathbb{K}^n$, where $\mathbb{K}$ is a number field.	4	2	2	4	1-3,12	10, first online WebWork task, problems 17-24 from the first midterm grading task	Scores for the work at the board and in class room
4	Matrix of a linear mapping. Matrix of a composite mapping. Transformation of the matrix of a linear mapping under a change of bases in the domain and in the codomain of a linear mapping. The problem	4	2	2	4	1-3,12	10, first online WebWork task, problems 25-33 from the first midterm	he work at the board and in class room

	of bringing of the matrix of a linear mapping to an almost diagonal form. Theorem on the sum of dimensions of the kernel and the image of a linear mapping. Calculation of the kernel and the image of a linear mapping. Finding a pair of bases diagonalizing the matrix of a linear mapping.						grading task	
Semedster 2, Module 2								
5	Linear operators. Injectivity, surjectivity, and bijectivity in the case of linear operators. Matrix of a linear operator. Transformation of the matrix of a linear operator under a change of a basis. Determinant of a linear operator. Non-degeneracy and bijectivity. Algebraic operations with linear operators and their propertes. Commutator and anticommutator of linear operators. Invariant subspaces of a linear operator. Sums and intersections of invariant subspaces. One-dimensional invariant subspaces and eigenvectors. Eigenvalues ant the characteristic equation of a linear operator. Roots of the characteristic equation (characteristic numbers). Simple and multiple characteristic numbers. The problem of bringing a matrix of a linear operator to some canonic form. Diaginalizable operators. Eigenspaces. Theorem on the sum of eigenspaces corresponding to distinct eigenvalues.	4	2	2	4	1-3,12	10, second online WebWork task, problems 1-12 from the first midterm grading task	Scores for the work at the board and in class room
6	Multiple eigenvalues and root subspaces of a linear operator. Two theorems on the sum of root subspaces corresponding to distinct eigenvalues. Peculiarities of the real and complex cases. Chains of vectors in root subspsaces. Initial and terminal vectors of a chain. Therem on linear independence of chains with linearly independent terminal vectors and theorem on a basis of chains in a root subspce.	4	2	2	4	1-3,12	10, second online WebWork task, problems 13-24 from the first midterm grading task	Scores for the work at the board and in class room

	Jordan blck, Jordan basis, and the Jordan normal form of the matrix of a linear operator. Hamilton-Cayley theorem. Linear functionals. Algebraic operations with functionals and the conjugate space. Coordinate functionals and the conjugate basis. The dimension of the conjugate space. Covectoral notation for linear functionals and he scalar product of a vector and a covector. Transformation of the coordinates of a covector under a change of a basis. Orthogonal complements of subspaces in the conjugate space and thedir dimensions. The properties of orthogonal complements. The conjugate mapping.							
7	Bilinear and quadratic forms. Symmetric bilinear forms. Recovering a symmetric bilinear form from the corresponding quadratic form. Components of bilinear and quadratic form in a basis. Transformation of the components of a quadratic form under a change of basis. The kernel of a quadratic form and orthogonal complements with respect to a quadratic form. Bringing the matrix of a quadratic form to the diagonal form. Zero inertia index and its relation with the dimwention of the kernel. Signatura of a quadratic form (peculiarities of the real and complex cases). Positive and negative inertia indices in the real case. Theorem on the invariantness of the positive and negative inertia indices. Positive and negative quadratic forms. Silvester criterion for positivity of a quadratic form.	4	2	2	4	1-3,12	10, second online WebWork task, problems 25-36 from the first midterm grading task	Scores for the work at the board and in class room
8	Positive quadratic forms as scalar products. Euclidean spaces. The triangle inequality and the Cauchy-Bunyakovsky-Schwarz inequality. The concept of the length of a vector and the concept of	4	2	2	6,5	1-3,12	10, second online WebWork task, problems 37-48 from the first midterm	Scores for the work at the board and in class room

	the angle between vectors. Gram matriz and its determinant. Theorem on complementing an orthonormal basis of a subspace up to an orthonormal basis in a whole space. Quadratic forms in a Euclidean spsace. Boundednes of quadratic forms in finite-dimensional Euclidean spaces and their norms. Extremal vectors and diagonalization of quadratic forms in an orthonoral basis. Diagonalization of a pair of forms one of which is positive.						grading task	
	Total hours for Semester 2:	**32**	**16**	**16**	**34,5**			
	Total hours for the year:	**66**	**36**	**0**	**129,6**			

Appendix 2

Rating-plan of the isciline Linear algebra and applications
Course of training (speciality) *01.03.02 Applied mathematics and informatics*
Year 1 , semester 1 (fall)

Forms of learning activities of students	Scores for each task	Number of tasks in a module	Scores	
			Minimum	Maximum
Module 1				
Current grading				
1. Work at the board and in class room			0	20
Midterm grading				
1. Solution of 34 online problems from the first task in WebWork	15	1	0	15
Module 2				
Current grading				
1. Work at the board and in class room			0	20
Midterm grading				
1. Solution of 34 online problems from the first task in WebWork	15	1	0	15
Rewarding scores				
According to the Regulation on the modular scoring system			0	10
Attending/missing classes (scores for missing classes are subtracted)				
Attending lectures			0	-6
Attending seminars			0	-10
Final grading				
1. Exam	15	2	0	30

Rating-plan of the isciline Linear algebra and applications
Course of training (speciality) *01.03.02 Applied mathematics and informatics*
Year __1__, semester __2 (spring)__

Forms of learning activities of students	Scores for each task	Number of tasks in a module	Scores	
			Minimum	Maximum
Module 1				
Current grading				
2. Work at the board and in class room			0	20
Midterm grading				
1. Solution of 33 online problems from the first task in WebWork	15	1	0	15
Module 2				
Current grading				
1. Work at the board and in class room			0	20
Midterm grading				
1. Solution of 48 online problems from the first task in WebWork	15	1	0	15
Rewarding scores				
According to the Regulation on the modular scoring system			0	10
Attending/missing classes (scores for missing classes are subtracted)				
Attending lectures			0	-6
Attending seminars			0	-10
Final grading				
1. Exam	15	2	0	30

Appendix 3

Examples of problems from the midterm grading tasks (semester 1)

From the first task in WebWork

Problem 1.1. Determine if the following matrices are in echelon form, are in reduced echelon form, or are not in echelon form:

$$\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -4 \end{bmatrix}, \qquad \begin{bmatrix} 4 & 0 & 1 \\ 0 & -8 & 0 \end{bmatrix},$$

$$\begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Problem 1.2. Bring the matrix $\begin{bmatrix} 3 & -1 & 2 & 8 \\ -2 & -3 & 2 & -2 \\ -2 & 1 & 3 & -1 \end{bmatrix}$ to a reduced echelon form.

From the second task in WebWork

Problem 2.3. Two vectors are given: $\mathbf{u} = \begin{bmatrix} -2 \\ 3 \\ -4 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 6 \\ 0 \end{bmatrix}$ Calculate the lengthes of these vectors and their scalar product.

Problem 2.4. Find the equation of a plane which is parallel to the plane $9\,x-7\,y-2\,z = -6$ and comprises the point $(-4,-5,-2)$. Write your answer as $a\,x+b\,y+c\,z = d$, where $a = 9$.

Problem 2.5. Find the equation of a plane which is perpendicular to the line

$$\begin{bmatrix} \mathrm{x} \\ \mathrm{y} \\ \mathrm{x} \end{bmatrix} = \begin{bmatrix} 8 \\ -4 \\ -5 \end{bmatrix} + \begin{bmatrix} 10 \\ -10 \\ 6 \end{bmatrix} t$$

and which comprises the point $(-4,-5,8)$. Write your answer as $a\,x + b\,y + c\,z = d$, where $a = 10$.

Problem 2.6. A plane is drawn throug the following three points $(-3,-1,0)$, $(-7,-4,-1)$, $(-7,-3,1)$. Find the normal vector to this plane.

Problem 2.7. Calculate the vector product $[\mathbf{a},\mathbf{b}]$ if $\mathbf{a} = \begin{bmatrix} -4 \\ 1 \\ 4 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 5 \\ -4 \\ 0 \end{bmatrix}$. Calculate the vector product $[\mathbf{c},\mathbf{d}]$ if $\mathbf{c} = 3\,\mathbf{e}_1 - 5\,\mathbf{e}_2 - 2\,\mathbf{e}_3$ and $\mathbf{d} = 1\,\mathbf{e}_1 - 4\,\mathbf{e}_2 + 0\,\mathbf{e}_3$, where $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ are vectors of some orthonormal basis in $\mathbb{R}^3$.

Examples of problems from the midterm grading tasks (semester 2)

From the first task in WebWork

Problem 1.1. Which of the subsets of $\mathbb{R}^{3\times 3}$ listed below are linear subspaces in $\mathbb{R}^{3\times 3}$?

- A) 3×3 matrices in a reduced echelon form.
- B) Diagonal 3×3 matrices.
- C) 3×3 matrices with zeros in the second row.
- D) Simmetric 3×3 matrices.
- E) Invertible 3×3 matrices.
- F) 3×3 matrices with integer entries.

Problem 1.2. Let x, y, z be some vectors and assume that $z = -1\,x - 3\,y$ and $w = -2\,x + 3\,y - 2\,z$. Wich of the following equalities are valid?

- A) $\mathrm{Span}(y) = \mathrm{Span}(w)$;
- B) $\mathrm{Span}(x, y) = \mathrm{Span}(x, w, z)$;
- C) $\mathrm{Span}(y, w) = \mathrm{Span}(z)$;
- D) $\mathrm{Span}(x, z) = \mathrm{Span}(y, w)$.

Problem 1.3. Let S_1 be the subspace of the space $M_4(\mathbb{R})$ composed by symmetric matrices. Let S_2 be the subspace of the space $M_5(\mathbb{R})$ composed by skew-symmetric matrices. Find the dimensions of these subspaces.

From the second task in WebWork

Problem 2.1. Calculate the determinant of the complex matrix

$$A = \begin{bmatrix} 3-i & 4+2i \\ 1+2i & -1-4i \end{bmatrix}.$$

Problem 2.2. Let

$$A = \begin{bmatrix} \text{-13} & 12 \\ \text{-16} & 15 \end{bmatrix}.$$

Find two different diagonal matrices D and the corresponding matrices S such that $A = SDS^{-1}$.

Appendix 4

An example of examination tickets (semester 1)

FEDERAL STATE BUDGETARY EDUCATIONAL INSTITUTION
OF HIGHER EDUCATION «BASHKIR STATE UNIVERSITY»
FACULTY OF MATHEMATICS AND INFORMATION TECHNOLOGIES
DEPARTMENT OF HIGHER ALGEBRA AND GEOMETRY

examination ticket № number is hidden
on «Linear algebra and applications, sem. 1» (20__-__ ac. year)

1. Systems of linear equations. Gauss's method for solving them.
2. Equations of a plane in the space.

Instructor __________ / Sharipov R. A. /

Head of the dept. __________ / Khabibullin B. N. /

An example of examination tickets (semester 2)

FEDERAL STATE BUDGETARY EDUCATIONAL INSTITUTION
OF HIGHER EDUCATION «BASHKIR STATE UNIVERSITY»
FACULTY OF MATHEMATICS AND INFORMATION TECHNOLOGIES
DEPARTMENT OF HIGHER ALGEBRA AND GEOMETRY

examination ticket № number is hidden
on «Linear algebra and applications, sem. 2» (20__-__ ac. year)

1. Complex numbers.
2. Root subspaces, chain bases and Jordan form of the matrix of a linear operator.

Instructor __________ / Sharipov R. A. /

Head of the dept. __________ / Khabibullin B. N. /