Ministry of Education and Science of Russia
federal state educational institution of higher education
"BASHKIR STATE UNIVERSITY"
FACULTY OF MATHEMATICS AND INFORMATION TECHNOLOGIES

Approved: at a meeting of the department
Protocol No. 7 dated 26.01. 2021
Head Department _______ /Yumagulov M.G.

agreed: Chairman of the educational and methodological management at Faculty of Mathematics and Information Technology
_________/Efimov A.M.

WORKING PROGRAM OF THE DISCIPLINE (MODULE)

discipline Differential equations

________________Mandatory part____________________

bachelors program: Direction of training (specialty)

01.03.02 Applied Mathematics and Informatics

Direction (profile) of training

Applied programming and data analysis
____

Qualification

bachelor

Developers (compilers) Associate Professor, Ph.D.	___________________/ __Nazirova E.A._

date of receipt: 2021

Ufa 2021 г

List of documents and materials

1. The list of planned learning outcomes in the discipline, correlated with the planned results of mastering the educational program
2. The purpose and place of discipline in the structure of the educational program
3. The content of the work program (volume of discipline, types and types of training sessions, educational and methodological support for independent work of students)
4. Fund of evaluation funds by discipline
   4.1 A list of competencies indicating the stages of their formation in the process of mastering the educational program. Description of indicators of criteria for assessing competencies at various stages of informing, description of assessment scales
   4. 2. Typical control tasks or other materials necessary for assessing knowledge, skills, abilities and experience of activity that characterize the stages of formation of competencies in the process of mastering the educational program.
5. Methodological materials that define the procedures for assessing knowledge, skills, and experience of activity that characterize the stages of the formation of competencies Educational, methodological and information support of the discipline
   5. 1. List of basic and additional educational literature necessary for mastering the discipline
   5. 2. List of resources of the information and telecommunication network "Internet" and software necessary for mastering the discipline, including professional databases and information support for the discipline
6. Material and technical base necessary for the implementation of the educational process in the discipline

1. **The list of planned learning outcomes in the discipline, correlated with the planned results of mastering the educational program**

The process of mastering the discipline (module) is aimed at the formation of the following competencies:

Category (group) of competencies	Competence being formed (with code)	Code and name of achievement indicator	Learning outcomes by discipline
1	2	3	4
Theoretical and practical foundations of professional activity	general professional competencies-1. Able to apply fundamental knowledge obtained in the field of mathematical and (or) natural sciences and use them in professional activities	general professional competencies-1.1. Possesses fundamental knowledge obtained in the field of mathematical or natural sciences	Knows the basic concepts, definitions and properties of objects of the taught discipline, formulations and proofs of statements, applications to other areas of mathematical knowledge and to disciplines of natural science content
		general professional competencies-1.2. Able to use fundamental knowledge gained in the field of mathematical and (or) natural sciences in professional activities	Able to prove statements and solve problems of the discipline taught, apply the acquired skills in other areas of mathematical knowledge, disciplines of natural science content
		general professional competencies-1.3. Has the skills to choose methods for solving problems of professional activity based on theoretical knowledge.	Has the skills to apply fundamental knowledge in the field of the discipline taught in future professional activities

2. The purpose and place of the discipline in the structure of the educational program.

Discipline "Differential Equations" refers to the mandatory part. The discipline is studied in the 2nd year in 3-4 semesters

The purpose of studying the discipline: the basic concepts of the theory of differential equations, the main types of differential equations and methods for their integration, apply general methods to solve specific problems in mathematics and physics. To master the discipline, competencies are required, formed in the framework of the study of the following disciplines: analytical geometry, mathematical analysis, algebra. Mastering the discipline "Differential Equations" is necessary for the subsequent study of the disciplines "Partial Differential Equations", "Probability Theory" and a number of others.

3. The content of the work program (volume of discipline, types and types of training sessions, educational and methodological support for independent work of students)

The content of the work program is presented in Appendix No. 1.

4. Fund of evaluation funds by discipline

4. 1. A list of competencies indicating the stages of their formation in the process of mastering the educational program. Description of indicators and criteria for assessing competencies at various stages of their formation, description of assessment scales

Code and wording of competence

General professional competencies-1. Able to apply fundamental knowledge obtained in the field of mathematical and (or) natural sciences and use them in professional activities

Code and name of the indicator of achievement of competence	**Learning Outcomes by discipline**	Criteria for evaluating learning outcomes			
		"Not Satisfactory"	«Satisfactory"		
		2 ("Not Satisfactory")	3 ("Satisfactory")	4 («Good»)	5 («Excellent»)
General professional competencies-1.1. Possesses fundamental knowledge obtained in the field of mathematical and (or) natural sciences	Knows the basic concepts, definitions and properties of objects of the taught discipline, formulations and proofs of statements, applications to other areas of mathematical knowledge and to disciplines of natural science content	Fragmentary ideas about the basic concepts, definitions and properties of objects of the taught discipline, formulations and proofs of statements, applications to other areas of mathematical knowledge and to disciplines of natural science	Incomplete ideas about the basic concepts, definitions and properties of objects of the discipline being taught, formulations and proofs of statements, applications to other areas of mathematical knowledge and to disciplines of natural science content	Formed, but containing separate gaps, ideas about the basic concepts x, definitions and properties x of objects of the taught discipline, formulations and proofs of statements, applications to other areas of mathematical knowledge and to disciplines of natural science content	Formed systematic ideas about the basic concepts, definitions and properties of objects of the discipline taught, formulations and proofs of statements, applications to other areas of mathematical knowledge and to disciplines of natural science content

		content			
General professional competencies-1.2. Усan use fundamental knowledge obtained in the field of mathematical and (or) natural sciences in professional activities	Able to prove statements and solve problems of the discipline taught, apply the acquired skills in other areas of mathematical knowledge, disciplines of natural science content	Fragmentary ability to apply fundamental knowledge in the field of the taught discipline in future professional activities	In general, successful, but not systematic, ability to apply fundamental knowledge in the field of the discipline taught in future professional activities	In general, successful, but containing some gaps, the ability to apply fundamental knowledge in the field of the taught discipline in future professional activities	Formed ability to apply fundamental knowledge in the field of the discipline taught in future professional activities
General professional competencies-1.3. Has the skills to choose methods for solving problems of professional activity based on theoretical knowledge	Has the skills to apply fundamental knowledge in the field of the discipline taught in future professional activities	partial possession of the skills to apply fundamental knowledge in the field of the taught discipline in future professional activities	In general, successful, but not systematic, application of the skills to apply fundamental knowledge in the field of the taught discipline in future professional activities	In general, successful, but containing some gaps, application of the skills to apply fundamental knowledge in the field of the taught discipline in future professional activities	Successful and systematic application of the skills of applying fundamental knowledge in the field of the taught discipline in future professional activities

The assessment criteria are the points that are set by the teacher for the types of activities (assessment tools) based on the results of studying the modules (sections of the discipline) listed in the rating plan of the discipline (for the exam: current control - a maximum of 40 points; midterm control - a maximum of 30 points, incentive points - maximum 10; for credit: current control - maximum 50 points; boundary control - maximum 50 points, incentive points - maximum 10).

Grading scales:

(for exam:

from 45 to 59 points - "satisfactory";

from 60 to 79 points - "good";

from 80 points - "excellent".

for offset:

credited - from 60 to 110 rating points (including 10 incentive points),

not credited - from 0 to 59 rating points).

4. 2. Standard control tasks or other materials necessary to assess knowledge, skills, abilities and experience that characterize the stages of formation of competencies in the process of mastering the educational program. Methodological materials that define the procedures for assessing knowledge, skills, and experience of activity that characterize the stages of the formation of competencies

Код и наименование индикатора достижения компетенции	Learning outcomes by discipline	Evaluation tools
General professional competencies-1.1.Possesses fundamental knowledge obtained in the field of mathematical and (or) natural sciences	Knows the basic concepts, definitions and properties of objects of the taught discipline, formulations and proofs of statements, applications to other areas of mathematical knowledge and to disciplines of natural science content	Test, settlement and graphic work, colloquium, test, exam
General professional competencies-1.2. Able to use fundamental knowledge gained in the field of mathematical and (or) natural sciences in professional activities	Able to prove statements and solve problems of the discipline taught, apply the acquired skills in other areas of mathematical knowledge, disciplines of natural science content	Test, settlement and graphic work, colloquium, test, exam
General professional competencies-1.3.Has the skills to choose methods for solving problems of professional activity based on theoretical knowledge	Has the skills to apply fundamental knowledge in the field of the discipline taught in future professional activities	Test, settlement and graphic work, colloquium, test, exam

Ranking plan discipline

The rating plan of the discipline is presented in Appendix 2

.Exam tickets

Exam ticket structure:
2 theoretical questions.
The first question for the 1st semester, the second question for the 2nd semester.

Exam questions:

1. The concept of a differential equation and its solution. integral curves. The concept of a general solution of a differential equation.
2. Cauchy problem. Theorem (Peano) of the existence of a solution to the Cauchy problem. Theorem (Picard) of existence and uniqueness of the solution of the Cauchy problem.
3. Differential equations integrable in quadratures. Equations with separable variables. Homogeneous equations of the form $x' = f(x, t)$. Уequations in total differentials of the form $P(x, t)dx + Q(x, t)dt = 0$. Linear equations of the first order of the form $x' = a(t)x + b(t)$.
4. Лlinear differential equations of the nth order of the form $x^{(n)} + a_1(t)x^{(n-1)} + + a_2(t)x^{(n-2)} + ... + a_{n-1}(t)x' + a_n(t)x = f(t)$. Fundamental system of solutions of a homogeneous linear equation.
5. General solution of a homogeneous and inhomogeneous linear equation of the nth order. Method of variation of arbitrary constants.
6. Scheme for solving second-order linear differential equations with constant coefficients $x'' + ax' + bx = f(t)$.
7. Matrix functions and their calculation. Matrix exponent e^{At}.
8. Linear systems of differential equations $x' = A(t)x + f(t)$. Fundamental decision system and fundamental decision matrix of a homogeneous system $x' = A(t)x$.
9. The Cauchy formula for the general solution of the homogeneous $x' = Ax$ and heterogeneous system $x' = Ax + f(t)$ differential equations with constant coefficients.
10. Formulas for the general solution of a linear homogeneous system of the second order (in terms of eigenvalues and eigenvectors of the matrix A).
11. Boundary Value Problems for Differential Equations. Solvability of boundary value problems for second-order linear differential equations.
12. The problem of rod bending.
13. Autonomous equations and systems. Properties of autonomous systems. Trajectories of autonomous systems and their difference from integral curves.
14. Equilibrium points (singular points) and periodic solutions (cycles) of autonomous systems.
15. Phase spaces and phase portraits of autonomous systems. Phase velocity field.
16. Phase portraits of autonomous equations of the first order.
17. Phase portraits of linear autonomous systems of the second order. Classification of singular points on the plane: node, saddle, focus, center.
18. Phase portraits of nonlinear autonomous systems of the second order in the neighborhood of a singular point. Linearized equation.
19. Phase portrait of a mathematical pendulum.

20. The concept of Lyapunov stability of solutions of differential equations. asymptotic stability. Stability properties of the zero solution of a linear equation я $x' = qx$ at $q < 0, q > 0$ and $q = 0$.
21. Signs of stability of the zero point of equilibrium of linear autonomous systems $x' = Ax$.
22. Signs of Stability of Equilibrium Points of Nonlinear Autonomous Systems $x' = f(x)$.
23. Stability of equilibrium points of a mathematical pendulum.
24. Stable matrices and polynomials. Stodol's theorem. Routh-Hurwitz criterion.
25. Fundamentals of numerical methods for solving the Cauchy problem. Euler method and Runge-Kutta method.
26. The concept of a dynamic system. Examples of dynamical systems: Malthus model, Verhulst model, predator-prey model, mathematical pendulum, Lorenz model. Equilibrium points of these systems and stability properties.

Exam Ticket Sample:

Ministry of Education and Science of Russia
federal state educational institution of higher education
"BASHKIR STATE UNIVERSITY"
FACULTY OF MATHEMATICS AND INFORMATION TECHNOLOGIES
DEPARTMENT OF DIFFERENTIAL EQUATIONS

Examination ticket No. 1 on the course "Differential Equations"

1. The concept of a differential equation and its solution. integral curves. The concept of a general solution of a differential equation.
2. Signs of Stability of Equilibrium Points of Nonlinear Autonomous Systems $x' = f(x)$.

Teacher Nazirova E.A../_______________/

Head of the Department Yumagulov M. G. /_______________/

The transfer of the score from 100-point to four-point is carried out as follows:

- excellent - from 80 to 110 points (including 10 incentive points);
- good - from 60 to 79 points;
- satisfactory - from 45 to 59 points;
- unsatisfactory - less than 45 points.

Evaluation criteria (in points):

- 25-30 points are given to the student if the student gave complete, detailed answers to all theoretical questions of the ticket, demonstrated knowledge of functionality, terminology, basic elements, the ability to apply theoretical knowledge in performing practical tasks. The student answered all additional questions without difficulty. The practical part of the work was completed completely without inaccuracies and errors;

- 17-24 points is given to a student if the student has revealed mostly theoretical questions, but there are inaccuracies in the definition of basic concepts. When answering additional questions, minor inaccuracies were made. When performing the practical part of the work, minor errors were made;

- 10-16 points are assigned to a student if, when answering theoretical questions, the student made several significant errors in the interpretation of basic concepts. The logic and completeness of the answer suffer from noticeable flaws. Gaps in knowledge of the basic methods are noticeable. On the whole, the theoretical questions are presented sufficiently, but with material omissions. There are fundamental errors in the logic of constructing the answer to the question. The student did not solve the problem or serious mistakes were made in solving the problem;

- 1-10 points are given to a student if the answer to theoretical questions indicates a misunderstanding and an extremely incomplete knowledge of the basic concepts and methods. There is a lack of skills in applying theoretical knowledge when performing practical tasks. The student was unable to answer any of the additional questions.

Sample Practice Topics

First order differential equations solved with respect to the derivative

1). The concept of a differential equation. Isocline method
2) Equations with separable variables;
3) Geometric and physical problems
4) Homogeneous equations;
5) Equations in total differentials; integrating factor;
6) Linear equation; Bernoulli equation; Differential equations not solved with respect to the derivative: :
7) Parameter introduction method;
8) Lagrange and Clairaut equations;
9) The existence and uniqueness theorem for the solution of the Cauchy problem; special solutions.
Higher order differential equations:
10) Equations that allow order reduction;
11) Linear equations with constant coefficients;
12) Linear equations with variable coefficients
13) Boundary value problems; Green's functions; Systems of ordinary differential equations:
14) Linear systems with constant coefficients;
15) General solution of a linear inhomogeneous system of equations; Sustainability :
16) Definition of stability according to Lyapunov, asymptotic stability; Lyapunov functions; sufficient conditions for asymptotic stability; stability in the first approximation;
17) Special points: saddle, node, focus, center;
18) Phase plane

Settlement and graphic works (SGW).

Description of SGW In each semester, the student is presented with two settlement and graphic works (SGW).

SGW No. 1 consists of 12 tasks.

SGW No. 2 consists of 9 tasks.

When performing the SGW, the student must be guided by the following instructions:

1. The work must be done on A4 sheets; the first page is the title page, which indicates the last name and first name of the student, group, option number.
2. The solution of problems should be given in the order of the numbers indicated in the SGW. Before solving each problem, it is necessary to completely rewrite its condition.
3. The solution of problems should be stated in detail, making appropriate references to information from the theory, indicating the necessary formulas and theorems.
4. The solution of problems of geometric content (phase portraits, etc.) must be accompanied by appropriate drawings.
5. The student selects the number of options in accordance with the number of his last name in the group list

SGW№1

1. Check (substitution) whether a function is $x = \frac{t^2 + t\ln ln(2t)}{t+1}$ solution of this differential equation $t(t + 1)(x' - 1) = x$. If yes, then indicate the domain of existence of the solution.
2. Using the isocline method, approximately draw the integral curves of the equation $y' = xy$ Find a general solution
3. For the differential equation $x = \sqrt{|x - 1|}\, lnt + lnx$:- determine the domain of existence of a solution to the Cauchy problem (determined by the conditions of the Piano theorem);
4. Compose a differential equation for curves that have the property that the tangent segment enclosed between the coordinate axes has length a.
5. How long does it take for half of the water to flow out of a cylindrical tank with a base diameter of 1 meter and a height of 1 meter through a hole 2 cm in diameter in the bottom? The axis of the cylinder is vertical, at the initial moments the tank is filled with water. Assume that water flows out of the hole at a speed equal to 3h meters/sec, where h is the height of the water level above the hole.
6. Find a general solution (or general integral) of the equation $(xy' - y)arctg\left(\frac{y}{x}\right) = x$
7. Find a general solution (or general integral) of the equation $(x^2 - y^2 - 4x)dx - 2xydy = 0$
8. Solve the Cauchy problem $x' + x = e^t$, $x(0) = 2$
9. Find a general solution (or general integral) of the equation $y = 2xy' - (y')^2$
10. Find a general solution (or general integral) of the equation $tx'' + x' = 2tx'$
11. .Find out if these functions sinx, cosx, cos (x + 1) form a linearly independent system
12. Show that the functions t+2, t^2-1 form a fundamental system of solutions of a linear homogeneous equation of the second order$(t^2 + 4t + 1)x'' - 2(t + 2)x' + 2x = 0$

Evaluation criterion SGW №1

credited if at least 8 tasks are correctly solved,
Not credited if less than 8 tasks are solved.

Evaluation criterion SGW №1

36 points are given to the student if 12 problems are solved correctly;

33 points are given to the student if 11 tasks are solved correctly;

30 points are given to the student if 10 problems are solved correctly.

27 points are given to the student if 9 problems are solved correctly

24 points are given to the student if 8 problems are solved correctly;

21 points are given to the student if 7 problems are solved correctly;

18 points are given to the student if 6 problems are solved correctly.

15 points are given to the student if 5 problems are solved correctly

12 points are given to the student if 4 tasks are solved correctly;

9 points are given to the student if 3 tasks are solved correctly

6 points are given to the student if 2 tasks are solved correctly;

3 points are given to the student if 1 problem is solved correctly;

SGW №2

1. Find a solution to the Cauchy problem $y'' + 2y' + y = 1,\ y(0) = 0,\ y'(0) = 2.$
2. Find out if the boundary value problem has a solution $y'' + 2y' + y = 1,\ y(0) = 0,\ y'(1) + y(1) = 0$
3. Calculate the matrix exponent e^At and construct a solution to the Cauchy problem $x' = Ax_0,\ x(0) = x_0,\ A = (-\ 1\ 2\ -\ 3\ 4\ )$
4. Find equilibrium points of first-order scalar equations$x' = x^3 - 2x^2 - x + 2$, $1 + \sin sin\, 2x = x'$, construct their phase portraits in the phase space and in the extended phase space.
5. Draw phase portraits of linear systems $x' = A_k x,\quad A_1 = [-\ 2\ -\ 1\ 1\ -\ 4\ ]A_2 = [-\ 1\ -\ 1\ 5\ 3\ ], A_3 = [4\ 1\ -\ 1\ 2\ ]$
6. Пapplying the Routh-Hurwitz criterion to find out for what values of the parameter a the zero solution of the equation $y^{(4)} + ay^{(3)} + 14y'' + 36y' + 45y = 0$ is asymptotically stable.
7. Find the equilibrium points of the system x'=f(x), find out the nature of their stability, depict schematically the phase portrait of the system in the vicinity of these points, give the corresponding linearized equations. $\{x_1' = x_2 - x_1\, x_2' = 2 - \sqrt{3x_1^2 + x_2^2}$
8. Consider the system $\{x' = (y - f(x))\, y' = x - y - z\, z' =- y$ withe $f(x) = \frac{x^3 - x}{6}$, Find the equilibrium points of the system, determine their nature of stability, and give the corresponding linearized equations.
9. Go from second order differential equation $y'' - \frac{(1-y^2)y'}{2} + y = 0$ to autonomous system $x' = F(x)\ x \in R^2$ by replacing $x_1 = y; x_2 = y'$ .Find the equilibrium points of the resulting system, determine their type, and find out the nature of its stability. Construct on the phase plane x1,x2 the trajectory of the solution x(t) of the resulting system on the interval 0≤t≤20 corresponding to the solution of the Cauchy problem for the differential equation.

SGW Criteria №2

Passed if at least 8 tasks are correctly solved,

Failed if less than 8 tasks are solved.

SGW Criteria №2

16 points are given to the student if 9 problems are solved correctly; points are given to the student if 8 tasks are solved correctly; points are given to the student if 7 problems are solved correctly. Points are given to the student if 6 problems are solved correctly points are given to the student if 5 problems are solved correctly; points are given to the student if 4 problems are solved correctly; points are given to the student if 3 tasks are solved correctly. points are given to the student if 2 tasks are solved correctly 1 point is given to the student if 1 problem is solved correctly;

Test papers

Description of the control work:
In each semester, the student is presented with two tests.
Each control work consists of 4-5 voluminous tasks.
The task is considered correctly solved if the student provides a detailed and complete solution.
Each task of control works No. 1,2 is estimated at 5 points.
Each task of control works No. 3,4 is estimated at 4 points**.**

Test No. 1

1. solve the equation $2(x\sqrt{y} + 1)ydx = xdy$,
2. solve the equation$(2xy^2 - 3y^3)dx + (y^3 - 3xy^2)dy = 0$
3. solve the equation $y\left(y - xy'\right) = \sqrt{x^4 + y^4}$
4. solve the equation $xy' = e^y + 2y'$

Evaluation criteria (in points)

20 points are given to the student if 4 tasks are solved correctly;
15 points are given to the student if 3 tasks are solved correctly;
10 points are given to the student if 2 problems are solved correctly.
5 points are given to the student if 1 problem is solved correctly

Test No. 2

1. solve the equation:$y'' - \frac{y'}{x-1} = x(x - 1)$.
2. solve the equation $y''(2y + 3) - 2(y')^2$ =0.
3. solve the equation xyy''-x$(y')^2 + y'\left(y' + y\right)$.
4. solve the equation $x(\left(y')^2 + e^2\right) =- 2y'$.

Evaluation criteria (in points)

20 points are given to the student if 4 tasks are solved correctly;
15 points are given to the student if 3 tasks are solved correctly;

10 points are given to the student if 2 problems are solved correctly.
5 points are given to the student if 1 problem is solved correctly

Test No. 3

1. solve the equation $y^{(6)} + 2y^{(5)} + y^{(4)} = 0$,
2. solve the equation $y'' + y = \frac{2}{sin^2 x}$,
3. solve the equation $y'' - 5y' = 2x - 3$,
4. solve the equation $x^2 y'' - 6xy' + 6y = x$,
5. solve the equation $x(x + 4)y'' - (2x + 4)y' + 2y = 0$.

.

Evaluation criteria (in points)

20 points are given to the student if all tasks are solved correctly;
16 points are given to the student if 4 tasks are solved correctly;
12 points are given to the student if 3 tasks are solved correctly;
8 points are given to the student if 2 tasks are solved correctly.
4 points are given to the student if 1 problem is solved correctly.

Test No. 4

1. Find the general solution of the system:

$$\begin{cases} \dot{x} = 5x + 2y + 7e^{-3t} \\ \dot{y} = -4x - 4y \end{cases}$$

2. Find the general solution of the system: $\{x' = x + z - y,\ y' = x + y - z\, z' = 2x - y$
3. Research for sustainability: $\{x' = 1 + \ln ln\,(1 + 2x) - e^y\, y' = 2x + tg(y)$

4. Explore Singular Points of an Equation $y' = \frac{-6x-5y}{x+3y}$

Evaluation criteria (in points)

20 points are given to the student if all tasks are solved correctly;
16 points are given to the student if 4 tasks are solved correctly;
12 points are given to the student if 3 tasks are solved correctly;
8 points are given to the student if 2 tasks are solved correctly.
4 points are given to the student if 1 problem is solved correctly.

Colloquium

Description of the colloquium

A colloquium is held every semester. The student is given the opportunity to answer one theoretical question from the list of questions for the colloquium.

Questions for Colloquium №1

1. Problems leading to differential equations. Examples.
2. The concept of a differential equation. Solution d.u. The main types of d. : nth order equation, first order equation, differential equation.
3. Geometric interpretation of solutions of d.u. Key words: integral curves, direction field, isoclines, method of isoclines for approximate construction of integral curves.
4. The Cauchy problem for a first-order differential equation. Solution of the Cauchy problem.
5. The (Peano) existence theorem for a solution to the Cauchy problem for a first-order differential equation. Formulation and examples, an example of non-uniqueness.
6. Theorem (Picard) of the existence and uniqueness of the solution of the Cauchy problem for a differential equation of the first order. Formulation and examples.
7. The concept of a general solution and a general integral of a first-order differential equation. Private decision. Examples.
8. Differential equations integrable in quadratures. Equations with separable variables.
9. Homogeneous equations.
10. Equations in total differentials.
11. Linear equations of the first order. General solution of a linear homogeneous equation.
12. General solution of a linear inhomogeneous equation of the first order. Bernoulli method (variations of arbitrary constants).
13. Bernoulli's equation and the scheme of its solution. Riccati equation.
14. Differential equations not resolved with respect to the derivative::

1) parameter introduction method;
2) Lagrange and Clairaut equations;
3) the existence and uniqueness theorem for the solution of the Cauchy problem;
4) special solutions.

Evaluation criteria (in points)

18-24 points are given to the student for a complete and detailed answer to a given question and for correctly given answers to additional questions.

- 11-17 points are given to a student for a complete and detailed answer to the question asked and did not answer 1-2 additional questions, or made an incomplete and / or fuzzy answer, but at the same time answered all additional questions.

- 1-10 points are given to the student if the student gave an incomplete answer to the question asked and did not answer one additional question. - 0 points are given to the student if the student did not answer the question and did not answer additional questions.

Questions for Colloquium №1

1. Linear differential equations of the nth order (LDE) of the form $x^{(n)} + a_1(t)x^{(n-1)} + a_2(t)x^{(n-2)} + \dots + a_{n-1}(t)x' + a_n(t)x = f(t)$. Fundamental system of solutions of a homogeneous LDE.

2. Theorems on the structure of the general solution of a homogeneous and inhomogeneous LDE. Method of variation of arbitrary constants.
3. LDE with constant coefficients. FSR and construction of a general solution of a homogeneous LDE on the example of a second-order equation x''+ax'+bx=0. Construction of a particular solution of an inhomogeneous LDE.
4. Linear systems of differential equations x'=Atx+f(t). Fundamental decision system and fundamental decision matrix of homogeneous system x'=Atx .
5. Cauchy's formula for the general solution of a homogeneous x'=Ax and an inhomogeneous system x'=Ax+f(t) of differential equations with constant coefficients.
6. Formulas for the general solution of a linear homogeneous system of the second order x'=Ax (in terms of eigenvalues and eigenvectors of the matrix A).
7. Boundary Value Problems for Differential Equations. Solvability of boundary value problems for second-order linear differential equations.
8. The problem of rod bending.
9. Autonomous equations and systems. Properties of autonomous systems. Trajectories of autonomous systems and their difference from integral curves.
10. Equilibrium points (singular points) and periodic solutions (cycles) of autonomous systems.
11. Phase spaces and phase portraits of autonomous systems. Phase velocity field.
12. Phase portraits of autonomous equations of the first order.
13. Phase portraits of linear autonomous systems of the second order x'=Ax.
14. Classification of singular points on the plane: node, saddle, focus, center.
15. Phase portraits of nonlinear autonomous systems of the second order in the neighborhood of a singular point. Linearized equation.
16. Phase portrait of a mathematical pendulum.
17. The concept of Lyapunov stability of solutions of differential equations. Asymptotic stability. Stability properties of the zero solution of the linear equation x'=qx for q<0, q>0 and q=0.
18. Signs of stability of the zero point of equilibrium of linear autonomous systems x'=Ax.
19. Signs of stability of equilibrium points of nonlinear autonomous systems x'=f(x).
20. Stability of equilibrium points of a mathematical pendulum.
21. Stable matrices and polynomials. Stodol's theorem. Routh-Hurwitz criterion.
22. Grids and grid functions. Function interpolation. Lagrange interpolation polynomial.
23. Fundamentals of numerical methods for solving the Cauchy problem. Approximate grid solution. Discretization error.
24. Euler method and Runge-Kutta method. One-step and multi-step methods.
25. Stability of linear periodic systems x'=Atx. Monodromy matrix. Multipliers.

Evaluation criteria (in points)

11-16 points are given to the student for a complete and detailed answer to a given question and for correctly given answers to additional questions.

-4-10 points is given to a student for a complete and detailed answer to a given question and did not answer 1-2 additional questions, or made an incomplete and / or fuzzy answer, but at the same time answered all additional questions.

- 1-4 points are given to the student if the student gave an incomplete answer to the question asked and did not answer one additional question. - 0 points are given to the student if the student did not answer the question and did not answer additional questions.

5. Educational, methodological and information support of the discipline

5. 1. List of basic and additional educational literature necessary for mastering the discipline

Main bibliography

1. 1. M. G. Yumagulov, Ordinary Differential Equations. Theory and applications. M.-Izhevsk: Izd-vo RHD, 2008. ISBN 978-5-93972-652-8. Access to the text is possible through the Electronic Library System (ELS) of BashSU, URL: https://bashedu. bibliotech. en

2. 2. M. G. Yumagulov, Introduction to the Theory of Dynamic Systems. - St. Petersburg. : Publishing house "Lan", 2015. - 272 p. ISBN 978-5-8114-1799-5. Access to the text is possible through the Electronic Library System (ELS) of BashSU, URL: https://bashedu. bibliotech. ruAvailable also through the electronic library system of the publishing house "Lan", URL: http://e. lanbook. com/.

3. 3. Alsevich, L. A. Differential Equations [Electronic resource]: workshop / Alsevich L. A. - Minsk: "Higher School", 2012 . — 384 p. – Access to the text of the electronic edition is possible through the Electronic Library System “University Library Online”. 2111-5.— <URL: http://www. biblioclub. ru/book/135999/>.

4. 4. Bibikov, Yu. N. Course of ordinary differential equations [Electronic resource]: tutorial / Yu. N. Bibikov. — Electronic data. - St. Petersburg: Lan, 2011. - 304 p. —Available through the Electronic Library System of the publishing house "Lan", Access mode: http://e. lanbook. com/.

5. 5. A. F. Filippov, Collection of Problems in Differential Equations. M.; Izhevsk: RHD Publishing House, 1998 (35 copies)

Additional literature:

1. Egorov, AI Classification of solutions of ordinary differential equations of the first order / AI Egorov. - Moscow: Fizmatlit, 2013. - 108 p. : ill. - Bibliography. : With. 105. - ISBN 978-5-9221-1489-9; The same [Electronic resource]. - URL: http://biblioclub. ru/index. php?page=book&id=275303
2. Rybakov, K. A. Ordinary differential equations: A practical course: a tutorial / K. A. Rybakov, A. S. Yakimova, A. V. Panteleev. - Moscow: Logos, 2010. - 384 p. - (New University Library). - ISBN 978-5-98704-465-0; The same [Electronic resource]. - URL: http://biblioclub. ru/index. php?page=book&id=84753

5. 2. List of resources of the information and telecommunications network "Internet" and software required for mastering the discipline

1. Windows 8 Russian. Windows Professional 8 Russian Upgrade. Agreement No. 104 dated 17.06.2013 Licenses are perpetual.

2. Microsoft Office Standard 2013 Russian. Agreement No. 114 dated November 12, 2014

Licenses are perpetual.

1	Electronic library system "EL BashGU"	Own electronic library of educational and scientific electronic publications, which includes publications of BashSU teachers	Authorized password access from anywhere on the Internet	Registration in the BashSU Library, further access from anywhere on the Internet	https://elib.bashedu.ru/
2	Electronic Library System "University Library Online"	Full-text database of educational and scientific electronic publications	Authorized password access from anywhere on the Internet	Registration in the BashSU Library, further access from anywhere on the Internet	http://www.biblioclub.ru/
3	Electronic library system of the publishing house "Lan"	Full-text database of educational and scientific electronic publications	Authorized password access from anywhere on the Internet	Registration in the BashSU Library, further access from anywhere on the Internet	http://e.lanbook.com/

6. Material and technical base necessary for the implementation of the educational process in the discipline

Name of special rooms and rooms for independent work	**Equipment of special rooms and rooms for independent work**	List of licensed software. Details of the supporting document
1. classroom for conducting lecture-type classes: classrooms No. 523, 515 (physics and mathematics building - educational). 2. classroom for conducting seminar-type classes: classrooms No. 511, 527, 522 (physics and mathematics building - educational). 3. classroom for group and individual consultations: classrooms No. 515, 523, 527, 522 (physics and mathematics building - educational). 4. classroom for current control and intermediate certification: classrooms No. 515, 523, 527, 522 (physics and mathematics building - educational). 5. rooms for independent work: auditorium No. 426 (Physical and Mathematical Building - Educational), Reading Room No. 2 (Physical and Mathematical Building - Educational)	Auditorium №511 Educational furniture, wall chalk board, multimedia projector Mitsubishi EX 320U 3D 2.4kg., screen on a tripod DraperDiplomat (1:1) 84/84* 213*213 MW, computer consisting of: system unit DEPO 460MD/3-540/T500G/ DVD-RW, monitor 20". Auditorium No. 515 Educational furniture, wall chalk board. Auditorium No. 523 Educational furniture, wall chalk board. Auditorium No. 527 Educational furniture, wall chalk board. Auditorium №522 Educational furniture, whiteboard, personal computer LenovoThinkCentre A70z IntelPentium E 5800, 320 Gb, 19" - 13 pcs., LessarLS/LU-H24KB2 air conditioner. Reading room №2 Educational furniture, educational and visual aids, fire safety stand, stationary monoblocks - 8 pcs, printer - 1 pc., scanner - 1 pc. Auditorium No. 426 Educational furniture, whiteboard, personal computers system unit /Core 15-7400 (3.0) / BGb/HDD1Tb/ 450W/Win 10 Pro/ USB keyboard. Mouse USB/LCD Monitor 21.5" – 14 pcs	1. Windows 8 Russian. Windows Professional 8 Russian Upgrade. Contract No. 104 dated June 17, 2013. Perpetual licenses. 2. Microsoft Office Standard 2013 Russian. Agreement No. 114 dated November 12, 2014. Perpetual licenses.

Application No. 1

Ministry of Education and Science of Russia
federal state educational institution of higher education
"BASHKIR STATE UNIVERSITY"
FACULTY OF MATHEMATICS AND INFORMATION TECHNOLOGIES

WORK PROGRAM CONTENT

disciplines Differential equations
Full-time education

Type of work	Volume of discipline
The total labor intensity of the discipline (ZET / hours)	6/216
Teaching hours for contact work with a teacher:	
lectures	52
practical / seminar laboratory	
laboratory	50
others (group, individual consultation and other types of educational activities involving the work of students with a teacher)	1,9
Teaching hours for independent work of students (SW)	59,3
Study hours to prepare for the exam / test / differentiated test (Control)	52,8

Forms of control:
Exam 4 semester
credit _3_ semester
SGW 4 semester

№ п/п	Theme and content	Form of studying materials: lectures, practical classes, seminars, laboratory work, independent work and labor intensity (in hours)				Obasic and additional literature recommended for students (numbers from the list)	Tasks for independent work of students	Form of current progress control (colloquia, tests, computer tests, etc.)
		Lecture	Pract	Pr	SW			
1	2	3	4	5	6	7	8	9
1.	introductory concepts. The concept of a differential equation and its solution. Geometric interpretation of solutions: integral curves, direction field, isoclines. System of differential equations. Cauchy problem. Existence and uniqueness theorems for the solution of the Cauchy problem. Continuous dependence of solutions on parameters and initial data. Special Solutions.	2	2		10, 3	**1, 4, 5, extra 4, 7**	problem solving [5] №1-6, № 15, №16 (a, б), № 17-20, №30, № 33, №36, № 36, № 37-45;	Test, SGW, colloquium, test, exam
2.	Differential equations integrable in quadratures. Elementary methods of integration. Separable equations, homogeneous equations, total differential equations, integrating factor, first order linear equation, Bernoulli and Riccati equations. Equations of Lagrange and Clairaut. Equations that allow order reduction.	16	16		12	**1, 4, 5, extra 4, 7**	problem solving [5] №425, №426, №427, №428, №429, №455, №456, №457, №458, №463, №464, №465, № 466, № 477, №481, №482, № 483, №241, №242, №243, №244, №245, №246, №251, №252, №253, №254, №255, №267, №268, №287,	Test, SGW, colloquium, test, exam

3.	Linear differential equations. Linear equations of the nth order; homogeneous and inhomogeneous equations. Cauchy problem. Fundamental system of solutions of a homogeneous equation. Vronsky's determinant. Theorems on the structure of the general solution of linear homogeneous and inhomogeneous equations. Method of variation of arbitrary constants. Linear differential equations of the nth order with constant coefficients. Boundary Value Problems for Second-Order Linear Differential Equations	12	12		12	**1, 4, 5, extra4, 7**	problem solving №501, №502 №511, №512, №513, №514, №515, №516, №549, №550, №551, №552, № 575, №576, №577, №578, № 582, № 583, № 584.	Test, SGW, colloquium, test, exam
4.	Systems of linear differential equations. Matrix functions; matrix exponent. Linear homogeneous and non-homogeneous systems of differential equations. Fundamental system of solutions and fundamental matrix of solutions of a homogeneous system. Formulas for the general solution of a linear system with constant coefficients. Method of variation of arbitrary constants.	10	12		12	**1, 4, 5, доп. 4, 7**	problem solving [5] №786, №787, №788, №789, №789, №790, №802, №803, №826, №828, № 829, №830, №846, №847, №848, №849	Test, SGW, colloquium, test, exam
5.	Autonomous equations and systems. Phase spaces and phase trajectories of autonomous systems. Phase velocity field. Equilibrium points (singular points) and periodic solutions. phase portraits. Phase portraits of autonomous equations of the first order and linear autonomous systems of the second order; classification of singular points: node, saddle, focus, center.	10	8		12	**1, 2, 3, 5, extra. 1-3**	№965, №967, №969, №971, №973, №979, №982, №984, №986, №988, №990[5]	Test, SGW, colloquium, test, exam
6	Sustainability. Lyapunov stability and asymptotic stability of solutions of differential equations. Sufficient conditions for the stability of equilibrium points of autonomous systems; stability in the first approximation. Routh-Hurwitz criterion. The concept of the Lyapunov function.	18	18		12	**1, 2, 3, 5, extra 1-3**	problem solving 5]№882, №883, №884, №885, №890, №891, №894, №899, №900, №907, №908, №914, №915, №916, №925, №	Test, SGW, colloquium, test, exam

							926, №932, №933, №934, №935	
7	Dynamic systems. The concept of a dynamic system. Examples: models of Malthus, Verhulst, Lotka-Volterra, Van der Pol, Lorenz. Attractors of dynamical systems.	2				**1, 2, 3, 5, extra 1-3**	problem solving 1. 2-1. 1. 6 главы 1 [2]	colloquium, test, exam,
	Total hours:	68	68		70, 3			

Application No. 2

Rating-plan of the discipline

Differential Equations

Direction of preparation _01. 03.02

Mathematics course ___2__________, semester ___3_____

Rating plan No. 1 (test)

Types of educational activities of students	Point for a specific task	Number of assignments per semester	Points	
			minimum	maximum
Module 1. First order differential equations				
current control				
SGW №1	3	12	0	36
Periodical control				
1.Written test №1	5	4	0	20
Module 2. Differential equations unresolved with respect to derivatives. Higher order differential equations				
current control				
Colloquium №1	24	1	0	24
Periodical control				
1. Written test №2	5	4	0	20
bonus points				
Student Olympiad or essay competition, publication of articles, solving problems of increased complexity			0	10
Attendance (points are deducted from the total points scored)				
1. Attending lectures			**0**	**–6**
2. Attending practical (seminar, laboratory classes)			**0**	**–10**
Final control				
1. Test				

Rating-plan of the discipline
Differential Equations
Direction of preparation _01. 03.02
Mathematics course ___2__________, semester ___4_____
Rating plan No. 2 (exam)

Types of educational activities of students	Point for a specific task	Number of assignments per semester	Points	
			minimum	maximum
Module 1 Linear differential equations. Systems of differential equations				
current control				
1. SGW №2	2	9	0	18
Periodical control				
1. Written test №3	4	5	0	20
Модуль 2. Stability. Phase portraits				
current control				
Colloquium №2	16	1	0	16
Periodical control				
1. Written test №4	4	4	0	16
bonus points				
Student Olympiad or essay competition, publication of articles, solving problems of increased complexity			0	10
Attendance (points are deducted from the total points scored)				
Attending lectures			**0**	**–6**
Attending practical (seminar, laboratory classes)			**0**	**–10**
Final control				
1. Test				
2. Exam			0	**30**