

BASHKIR STATE UNIVERSITY FACULTY OF MATHEMATICS AND  
INFORMATION TECHNOLOGY

Approved:  
at the meeting of the Department ,  
Protocol No. 7 of January 20, 2022.  
Head of the Department  
\_\_\_\_\_ / Yumagulov M.G.

Agreed:  
Chairman of the UMK FMIT  
\_\_\_\_\_ / Efimov A.M.

**WORKING PROGRAM OF THE DISCIPLINE (MODULE)**

discipline Partial differential equations  
(name of the discipline)

Mandatory part

(specify the part (mandatory part or part formed by participants of educational relations, optional)

**Bachelor's degree program**

Direction of training (specialty)

01.03.02 "Applied mathematics and informatics"

(the code and the name of the direction of training (specialty) are indicated)

The orientation (profile) of the training

"Mathematical modeling and computational mathematics"

(the name of the orientation (profile) of the training is indicated)

Qualification

Bachelor

(qualification is indicated)

Developers (compilers)	
Candidate of Physical and Mathematical Sciences, docent	_____/ N.A.Sidelnikova

For admission: 2022

Ufa 2022

Compiler / compilers: \_\_\_\_\_

The working program of the discipline was approved at the meeting of the department protocol from «\_\_\_\_\_» \_\_\_\_\_ 20\_\_ year № \_\_\_\_\_

Additions and changes made to the work program of the discipline were approved at the meeting of the department \_\_\_\_\_

\_\_\_\_\_ ,  
protocol № \_\_\_\_\_ от «\_\_\_\_\_» \_\_\_\_\_ 20 \_ year.

Head of the Department \_\_\_\_\_ / \_\_\_\_\_ F.N.P./

Additions and changes made to the work program of the discipline were approved at the meeting of the department \_\_\_\_\_

\_\_\_\_\_ ,  
protocol № \_\_\_\_\_ от «\_\_\_\_\_» \_\_\_\_\_ 20 \_ year.

Head of the Department \_\_\_\_\_ / \_\_\_\_\_ F.N.P./

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protocol № \_\_\_\_\_ от «\_\_\_\_\_» \_\_\_\_\_ 20 \_ year.

Head of the Department \_\_\_\_\_ / \_\_\_\_\_ F.N.P./

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protocol № \_\_\_\_\_ от «\_\_\_\_\_» \_\_\_\_\_ 20 \_ year.

Head of the Department \_\_\_\_\_ / \_\_\_\_\_ F.N.P./

### **List of documents and materials**

1. The list of planned learning outcomes in the discipline, correlated with the planned results of the development of the educational program.
2. The purpose and place of the discipline in the structure of the educational program.
3. The content of the work program (the scope of discipline, types and types of training sessions, educational and methodological support for independent work of students).
4. The fund of evaluation funds for the discipline.
  - 4.1. A list of competencies indicating the stages of their formation in the process of mastering the educational program. Description of indicators and criteria for assessing competencies at various stages of their formation, description of assessment scales.
  - 4.2. Standard control tasks or other materials necessary for the assessment of knowledge, skills, skills and experience of activities that characterize the stages of competence formation in the process of mastering the educational program. Methodological materials defining the procedures for assessing knowledge, skills, skills and experience of activities that characterize the stages of competence formation.
  - 4.3. Rating-discipline plan (if necessary)
1. Educational, methodological and informational support of the discipline
  - 5.1. The list of basic and additional educational literature necessary for the development of the discipline.
  - 5.2. The list of resources of the information and telecommunication network "Internet" and software necessary for the development of the discipline.
1. The material and technical base necessary for the implementation of the educational process in the discipline

**1. The list of planned learning outcomes in the discipline, correlated with the indicators of achievement of competencies established in the educational program**

According to the results of mastering the discipline, the student must achieve the following learning outcomes:

<b>Category (group) of competencies<sup>1</sup> (if there is a defense industry)</b>	<b>The competence being formed (with the indication of the code)</b>	<b>Code and name of the competence achievement indicator</b>	<b>Results of training in the discipline</b>
Systemic and critical thinking	OPK-1 -1 "The ability to solve standard tasks of professional activity on the basis of information and bibliographic culture with the use of information and communication technologies and taking into account the basic requirements of information security "	OPK-1.1. Algorithmizes problem solving and implements algorithms using software tools	Know important scientific results and problems of modern mathematics and related fields. Be able to develop new research methods and algorithms and apply them in independent research activities. Master the methods of construction and research of mathematical models
		OPK-1.2. Uses information technology tools to search, store, process, analyze and provide information	Be able to solve specific tasks based on the reproduction of logical and rhetorical means of analysis. Know the specific logical means of deductive and inductive reasoning and argumentation. Have the skills to solve atypical, analytical problems.
		OPK-1.3. Demonstrates knowledge of the requirements for graphic documentation.	Possess the skills of solving theoretical and practical problems with the help of modern software tools. Know the basic methods of obtaining research activities in the field of

<sup>1</sup> Specified only for the Criminal Code and the defense Industry (if available).

			mathematics. Be able to apply fundamental knowledge of mathematics and computer science to solve applied problems.
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**1. The purpose and place of the discipline in the structure of the educational program**

The discipline "Partial differential equations" refers to the mandatory part. The discipline is studied in the 3rd year in the 5th and 6th semesters. Objectives of the discipline: is an overview of some of the most common methods of research and solving basic classical problems for partial differential equations

**2. The content of the work program (the scope of discipline, types and types of training sessions, educational and methodological support for independent work of students)**

The content of the work program is presented in Appendix No. 1.

**1. The fund of evaluation funds for the discipline**

**4.1. List of competencies and indicators of achievement of competencies with indication of the planned learning outcomes related to them in the discipline. Description of criteria and scales for evaluating the results of training in the discipline.**

**Code and wording of the competence of the OPK-1** "The ability to solve standard tasks of professional activity on the basis of information and bibliographic culture with the use of information and communication technologies and taking into account the basic requirements of information security "

Code and name of the competence achievement indicator	Results of training in the discipline	Criteria for evaluating learning outcomes			
		2 ("Not satisfactory")	3 ("Satisfactory")	4 ("Good")	5 ("Excellent")
The first stage (level) Ability to solve standard tasks of professional activity on the basis of information and bibliographic culture with the use of information and communication technologies	To know: Z(OPK-1)-1 on the patterns of development of scientific and technological progress, the structure of local and global computer networks, the basic requirements of information security, general characteristic	Does not have a clear idea of the material being studied, makes gross mistakes	Fragmentary , incomplete knowledge without gross errors	Generally successful, but containing some gaps in the basic (standard) volume	Demonstration of a high level of knowledge, the ability to independently analyze and implement the acquired knowledge

<p>and taking into account the basic requirements of information security</p>	<p>s of the processes of collecting, transmitting and processing information, the current state and trends in the development of technical and software automation and computerization in the field of quality management</p>				
	<p>Be able to: U(OPK-1)-1 Apply mathematical apparatus to solve practical problems of professional activity, use computer technologies for planning, organizing and carrying out work on technical regulation and metrology, understand and solve professional tasks in the field of management and research and production activities in accordance with the profile of training</p>	<p>Does not have a clear idea of the material being studied, makes gross mistakes</p>	<p>Fragmentary, incomplete knowledge without gross errors</p>	<p>Generally successful, but containing some gaps in the basic (standard) volume</p>	<p>Demonstration of a high level of knowledge, the ability to independently analyze and implement the acquired knowledge</p>
	<p>Own:</p>	<p>Does not</p>	<p>Fragmentary</p>	<p>Generally</p>	<p>Demonstratio</p>

	<p>In(OPK-1)-1 The main methods of theoretical and experimental research, skills in using standard software tools, computer skills with graphic packages to obtain design, technical and other documents, skills in working with measuring instruments and devices for their interface with a computer as a means of control and information processing</p>	<p>have a clear idea of the material being studied, admits rough</p>	<p>, incomplete knowledge without gross errors</p>	<p>successful, but containing some gaps in the basic (standard) volume</p>	<p>n of a high level of knowledge, the ability to independently analyze and implement the acquired knowledge</p>
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Above is a table for the form of intermediate control – exam, for the test we specify the evaluation criteria for the scale: "Credited", "Not credited".

**4.2. Standard control tasks or other materials necessary for evaluating the results of training in the discipline, correlated with the indicators of competence achievement established in the educational program. Methodological materials defining the procedures for evaluating the results of training in the discipline.**

**The list of planned learning outcomes in the discipline, correlated with the planned results of the development of the educational program**  
(with a focus on competence maps)

**The content of the work program (the scope of the discipline, types and types of training sessions, educational and methodological support for independent work of students).**

The content of the work program is presented in Appendix No. 1.

## 1. The fund of evaluation funds for the discipline

### 4.1. A list of competencies indicating the stages of their formation in the process of mastering the educational program. Description of indicators and criteria for assessing competencies at various stages of their formation, description of assessment scales

Code and formulation of competence:

OPK-1: *The ability to solve standard tasks of professional activity on the basis of information and bibliographic culture with the use of information and communication technologies and taking into account the basic requirements of information security*

Stage (level) of competence development	Planned learning outcomes (indicators of achievement of a given level of competence development)	Criteria for evaluating learning outcomes			
		2 ("Not satisfactory")	3 ("Satisfactory")	4 ("Good")	5 ("Excellent")
The first stage (level)	<b>To know:</b> the content of the material on the subject, the main methods of solving problems, the main theorems of the discipline taught	Lack of knowledge	Partial knowledge of the content of the material on the subject, the main methods of solving problems, the main theorems of the discipline taught	Complete and clear, but containing separate gaps, knowledge of the content of the material on the subject, the main methods of solving problems, the main theorems of the discipline taught	Complete and clear knowledge of the content of the material on the subject, the main methods of solving problems, the main theorems of the discipline taught
Second stage (level)	<b>Be able to:</b> solve problems in the discipline taught, determine the correctness of the task, apply knowledge of the subject in practice	Lack of skills	Fragmentary skills to solve problems in the discipline taught, to determine the correctness of the task, to put into practice knowledge of the subject	In general, successful, but containing some gaps, the ability to solve problems in the discipline taught, to determine the	The formed ability to solve problems in the discipline taught, to determine the correctness of the task, to



				correctness of the task, to put into practice knowledge of the subject	put into practice knowledge of the subject
The third stage (level)	<b>Possess:</b> the ability to correctly set a task, classical modern methods of discipline, the conceptual apparatus of the subject	Lack of possessions	In general, successful, but not systematic possession of the ability to correctly set a task, classical modern methods of discipline, the conceptual apparatus of the subject	Generally successful, but containing some gaps in the ability to correctly set the task, classical modern methods of discipline, the conceptual apparatus of the subject	Successful possession of the ability to correctly set a task, classical modern methods of discipline, the conceptual apparatus of the subject

**Evaluation tools include:** *Individual, group survey; written answers to questions; oral survey (questions for self-control); laboratory work; control work; interview; report; message; task; practical assignment; abstract; tests; colloquium; report (on practices, research work of students, etc.); scientific report on the topic of research and development; case task; complex practical task, project; creative tasks (speeches, presentations, crossword preparation, etc.); essay; article; situational tasks; round table; debate; discussion; brainstorming; business, role-playing games; workbook; trainings; computer simulations, simulators; tasks using an interactive whiteboard, etc.*

Further, for full-time, part-time and part-time forms of bachelor's/specialist/master's education, the evaluation criteria and, if necessary, evaluation tools are described separately (taking into account the presence/absence of a modular rating system of assessments, control works for part-time students, etc.).

The evaluation criteria for the module-rating system are the points that are set by the teacher for the types of activities (evaluation tools) based on the results of studying the modules (sections of the discipline) listed in the discipline rating plan (for the exam: current control - a maximum of 40 points; boundary control – a maximum of 30 points, incentive points – a maximum of 10; for credits: current control – a maximum of 50 points; boundary control – a maximum of 50 points, incentive points – a maximum of 10).

Assessment scales:

*(for the exam:*

from 45 to 59 points – "satisfactory";

from 60 to 79 points – "good";

from 80 points – "excellent".

for the test:

credited – from 60 to 110 rating points (including 10 incentive points),

not credited – from 0 to 59 rating points).

**4.2. Standard control tasks or other materials necessary for the assessment of knowledge, skills, skills and experience of activities that characterize the stages of competence formation in the process of mastering the educational program. Methodological materials defining the procedures for assessing knowledge, skills, skills and experience of activities that characterize the stages of competence formation**

Stages of development	Learning outcomes	Competence	Evaluation tools
1st stage Knowledge	<b>To know:</b> the content of the material on the subject, the main methods of solving problems, the main theorems of the discipline taught	OPK-1: The ability to solve standard tasks of professional activity on the basis of information and bibliographic culture with the use of information and communication technologies and taking into account the basic requirements of information security	Control work, RGR, exam
2nd stage Skills	<b>Be able to:</b> solve problems in the discipline taught, determine the correctness of the task, apply knowledge of the subject in practice	OPK-1: The ability to solve standard tasks of professional activity on the basis of information and bibliographic culture with the use of information and communication technologies and taking into account the basic requirements of information security	Control work, RGR, exam
Stage 3 Possess skills	<b>Own:</b> the ability to correctly set the task, classical modern methods of discipline, the conceptual apparatus of the subject	OPK-1: The ability to solve standard tasks of professional activity on the basis of information and bibliographic culture with the use of information and communication technologies and taking into account the basic requirements of information security	Control work, RGR, exam

**1.3. Rating-discipline plan  
(if necessary)**

The rating plan of the discipline is presented in Appendix 2.

### **Exam tickets**

The structure of the examination ticket: the ticket consists of two theoretical questions.

Sample questions for the exam:

1. Reduction to the canonical form of second-order equations with two independent variables
2. The concept of the characteristic of a linear equation of the second order. The role of characteristics in the formulation of the Cauchy problem.
3. Definition of the Cauchy-Kovalevskaya system and formulation of the Cauchy problem for it
4. The Cauchy problem with analytical data. The Cauchy-Kovalevskaya theorem.
5. The concept of correctness of mathematical physics problems. Hadamard 's example
6. One-dimensional wave equation. The Dalembert formula.
7. A wave equation with three spatial variables. Kirchhoff Formula.
8. A wave equation with two spatial variables. The Poisson formula. Descent method
9. The concept of the area of dependence, the area of influence and the area of definition
10. Inhomogeneous wave equation. The case of three spatial variables. Lagging potential
11. Inhomogeneous wave equation the case of two and one spatial variables
12. Uniqueness of the solution of the Cauchy problem
13. Correctness of the statement of the Cauchy problem
14. Gursa's task
15. Incorrectly set tasks
16. The Riemann function
17. The equation of thermal conductivity. The maximum principle
18. The first boundary value problem for the heat equation
19. The Cauchy problem. Smoothness of the solution
20. Inhomogeneous equation of thermal conductivity
21. Harmonic functions. Properties of harmonic functions. The concept of a regular harmonic function at an infinitely distant point
22. Green's formulas for harmonic functions
23. Integral representation of harmonic functions
24. Mean theorems for harmonic functions. The principle of extremum
25. Green's function of the Dirichlet problem for the Laplace equation
26. Green's function of the Dirichlet problem for the Laplace equation in a ball. Poisson Formula
27. Green's function of the Dirichlet problem for the Laplace equation in a half-space. Poisson Formula
28. The main property of sign-constant harmonic functions. Liouville 's Theorem
29. Harnack's theorem for harmonic functions
30. Potentials of the simple and double layer. The simplest properties, formulas for the jump
31. Reduction of Dirichlet and Neumann problems to integral equations. Application of Fredholm theory to solving Dirichlet and Neumann problems
32. Determination of the generalized solution of the Dirichlet problem for the Laplace and Poisson equation.
33. Solution of a mixed problem for the string oscillation equation

34. General properties of eigenfunctions and eigenvalues
35. Justification of the Fourier method
36. Study of membrane vibrations.

Sample exam ticket

**Bashkir State University Faculty of Mathematics and Information Technology**

**Department of Differential Equations**

**Course of study 01.03.02 "Applied mathematics and informatics"**

discipline: *"Partial differential equations"*,

II (6) sem. 20\_\_ - \_\_ of the academic year

**Examination ticket No. 1**

1. Derivation of the membrane oscillation equation.
2. Harnack's theorem for harmonic functions.

Teacher Sidelnikova N.A. / \_\_\_\_\_/

Head of the Department Yumagulov M.G. / \_\_\_\_\_/

The transfer of an assessment from a 100-point to a four-point one is performed as follows:

- excellent – from 80 to 110 points (including 10 incentive points);
- good – from 60 to 79 points;
- satisfactory – from 45 to 59 points;
- unsatisfactory – less than 45 points.

Approximate criteria for evaluating the answer to the exam (only for those who study using a modular rating system of training and assessment of student performance):

**Evaluation criteria (in points):**

*- 25-30 points are awarded to the student if the student has given complete, detailed answers to all theoretical questions of the ticket, demonstrated knowledge of functionality, terminology, basic elements, the ability to apply theoretical knowledge when performing practical tasks. The student answered all the additional questions without difficulty. The practical part of the work is completely done without inaccuracies and errors;*

*- 17-24 points are awarded to the student if the student has revealed mainly theoretical questions, but inaccuracies in the definition of basic concepts are allowed. There are minor inaccuracies when*

answering additional questions. Minor mistakes were made when performing the practical part of the work;

- **10-16** points are awarded to the student if, when answering theoretical questions, the student made several significant mistakes in the interpretation of basic concepts. The logic and completeness of the answer suffer from noticeable flaws. There are noticeable gaps in the knowledge of the basic methods. The theoretical issues are generally presented sufficiently, but with omissions of material. There are fundamental errors in the logic of constructing the answer to the question. The student did not solve the problem or gross mistakes were made in solving it;

- **1-10** points are awarded to the student if the answer to theoretical questions indicates a misunderstanding and extremely incomplete knowledge of basic concepts and methods. There is a lack of skills in applying theoretical knowledge when performing practical tasks. The student could not answer any additional questions.

### Tasks for control work No. 1

1. Solve the Cauchy problem:

$$3u_x - 4u_y + e^{4x+3y}u = 0,$$

2. Solve the Cauchy problem:

$$3u_x + u_z - 2u_y = x + y,$$

### Tasks for control work No. 2

1. Solve the Cauchy problem by factorization:

$$3u_{xxx} + 2u_{xxy} - u_{xyy} - 2u_{yyy} = 0,$$

$$u(x, 0, z) = x(z - 1).$$

2. Determine the type of equation:

$$u_{xy} + u_{yz} + u_{xz} - 3x^2 u_y + y \sin x u + xe^{-y} = 0.$$

### Tasks for control work No. 3

1. Determine the type of system:

$$u_x + v + 7u_x - 2u_y = 0,$$

$$3u_x + 3v + 31u_x + v - e^y \sin x = 0.$$

2. Bring the equation to the canonical form:

$$u_{xx} - 2u_{xy} + u_{yy} + a u_x + b u_y + gu = 0.$$

### Tasks for control work No. 4

1. Formulate the problem of small longitudinal vibrations of an elastic homogeneous rod of variable cross-section  $S = S(x)$  of length  $l$  under arbitrary initial conditions for the cases

a) the rod has the shape of a truncated cone with radii of bases  $r$  and  $R$  ( $r < R$ ), which are fixed rigidly.

b) the end of the rod  $x = 0$  is fixed elastically, and the end  $x = l$ , starting from the moment  $t = 0$ , a longitudinal force  $F(t)$  is applied per unit cross-sectional area.

2. In the band  $0 < x < l$ ,  $t > 0$  for the equation  $utt = a^2 u_{xx}$ , solve a mixed problem with the

conditions:

$$u(0, t) = u(l, t) = 0,$$

$$u(x, 0) = 0, u(x, 0) = \sin \frac{2\pi}{l} x.$$

### Tasks for control work No. 5

1. In the band  $0 < x < l$ ,  $t > 0$  for the equation  $u_t = a^2 u_{xx}$ , solve a mixed problem with the

conditions:  $u(0, t) = u(l, t) = 0$ ,  $u(x, 0) = Ax$ .

2. Find the solution of the Laplace equation in the half-band  $0 < x < l$ ,  $0 < y < l$ , respectively, according to the boundary conditions

$$u(x, 0) = u_y(x, l) = 0, \quad u(0, y) = f(y), u(l, y) = 0.$$

Description of the assessment methodology:

**Evaluation criteria (in points):**

- **25-30** points are awarded to the student if the student has given complete, detailed answers to all theoretical questions of the ticket, demonstrated knowledge of functionality, terminology, basic elements, the ability to apply theoretical knowledge when performing practical tasks. The student answered all the additional questions without difficulty. The practical part of the work is completely done without inaccuracies and errors;
- **17-24** points are awarded to the student if the student has revealed mainly theoretical questions, but inaccuracies in the definition of basic concepts are allowed. There are minor inaccuracies when answering additional questions. Minor mistakes were made when performing the practical part of the work;
- **10-16** points are awarded to the student if, when answering theoretical questions, the student made several significant mistakes in the interpretation of basic concepts. The logic and completeness of the answer suffer from noticeable flaws. There are noticeable gaps in the knowledge of the basic methods. The theoretical issues are generally presented sufficiently, but with omissions of material. There are fundamental errors in the logic of constructing the answer to the question. The student did not solve the problem or gross mistakes were made in solving it;
- **1-10** points are awarded to the student if the answer to theoretical questions indicates a misunderstanding and extremely incomplete knowledge of basic concepts and methods. There is a lack of skills in applying theoretical knowledge when performing practical tasks. The student could not answer any additional questions.

### Tasks for RGR

In the 6th semester, the student is presented with one calculation and graphic work (RGR). Each RGR consists of 2 tasks. The problem is considered to be solved correctly if the student provides a detailed and complete solution to it.

**Task 1.** Determine the type of equations. Bring it to the canonical form.

1.  $u_{xx} + 2u_{xy} + 5u_{yy} - 32u_y = 0$ .
2.  $u_{xx} - 2u_{xy} + u_{yy} + 9u_x + 9u_y = 0$
3.  $2u_{xx} + 3u_{xy} + u_{yy} + 7u_x + 4u_y = 0$ .
4.  $u_{xx} + u_{xy} - 2u_{yy} - 3u_x - 15u_y + 27x = 0$ .
5.  $9u_{xx} - 6u_{xy} + u_{yy} + 10u_x - 15u_y + x - 2y = 0$ .
6.  $u_{xx} + 2u_{xy} + 10u_{yy} - 24u_x + 42u_y + 2(x + y) = 0$ .
7.  $u_{xx} + 4u_{xy} + 13u_{yy} + 3u_x + 24u_y + 9(x + y) = 0$ .
8.  $u_{xx} - 4u_{xy} + 5u_{yy} - 3u_x + u_y = 0$
9.  $u_{xx} - 6u_{xy} + 9u_{yy} - u_x + 2u_y = 0$ .
10.  $2u_{xy} - 4u_{yy} + u_x - 2u_y + x = 0$ .
11.  $u_{xy} + 2u_{yy} - u_x + 4u_y = 0$ .
12.  $2u_{xx} + 2u_{xy} + u_{yy} + 4u_x + 4u_y = 0$
13.  $u_{xx} + 2u_{xy} + u_{yy} + 3u_x - 5u_y = 0$ .
14.  $u_{xx} - u_{yy} + u_x + u_y = 0$ .
15.  $u_{xx} + u_{xy} - u_y + 4x = 0$ .
16.  $3u_{xx} + u_{xy} + 3u_x + u_y + y = 0$ .
17.  $u_{xx} + 4u_{xy} + 5u_{yy} - 2u_x - 2u_y = 0$ .
18.  $5u_{xx} + 16u_{xy} + 16u_{yy} + 24u_x + 32u_y = 0$ .
19.  $u_{xx} - 2u_{xy} + u_{yy} - 3u_x + 12u_y = 0$
20.  $y^2 u_{xx} + 2xyu_{xy} + x^2 u_{yy} = 0$

**Task 2.** Solve the following problem for the inhomogeneous wave equation by the method of separation of variables.

1.  $u_{tt} = a^2 u_{xx} + Ax + B$ ,  $0 < x < 1$ ,  $t > 0$ ,  
 $u(0, t) = U_1$ ,  $u(1, t) = U_2$ ,  
 $u(x, 0) = U_1(1 - l^{-1}x) + U_2 l^{-1}x$ ,  $u_t(x, 0) = 0$ ,  
 a)  $A = 2$ ,  $B = 1$ ,  $U_1 = 1$ ,  $U_2 = 0$ ,  
 б)  $A = 1$ ,  $B = 2$ ,  $U_1 = 0$ ,  $U_2 = 1$ ,  
 в)  $A = 1$ ,  $B = 0$ .
2.  $u_{tt} = a^2 u_{xx} + Ax + B$ ,  $0 < x < 1$ ,  $t > 0$ ,  
 $u_x(0, t) = 0$ ,  $u(1, t) = U$ ,  
 $u(x, 0) = U$ ,  $u_t(x, 0) = V$ ,  
 а)  $A = 2$ ,  $B = 1$ ,  $U = 1$ ,  $V = 0$   
 б)  $A = 3$ ,  $B = 1$ ,  $U = 2$ ,  $V = 1$   
 в)  $A = 1$ ,  $B = 0$ ,  $U = 1$ ,  $V = 2$ .
3.  $u_{tt} = a^2 u_{xx} + Ax + B$ ,  $0 < x < 1$ ,  $t > 0$ ,  
 $u(0, t) = U$ ,  $u_x(1, t) = 0$ ,  
 $u(x, 0) = U$ ,  $u_t(x, 0) = V$ ,  
 а)  $A = 2$ ,  $B = 1$ ,  $U = 1$ ,  $V = 0$   
 б)  $A = 4$ ,  $B = 1$ ,  $U = 2$ ,  $V = 1$   
 в)  $A = 1$ ,  $B = 0$ ,  $U = 1$ ,  $V = 2$ .
4.  $u_{tt} = a^2 u_{xx} + Ax + B$ ,  $0 < x < 1$ ,  $t > 0$ ,  
 $u_x(0, t) = u_x(1, t) = 0$ ,  
 $u(x, 0) = U$ ,  $u_t(x, 0) = V$ ,  
 а)  $A = 2$ ,  $B = 1$ ,  $U = 1$ ,  $V = 0$



$$\begin{aligned} \text{б) } A = 1, B = 1, U = 2, V = 1 \\ \text{в) } A = 1, B = 0, U = 1, V = 2. \end{aligned}$$

$$\begin{aligned} 5. \quad u_{tt} = a^2 u_{xx} + A \sin x + B, \quad 0 < x < 1, \quad t > 0, \\ u(0, t) = U_1, \quad u(1, t) = U_2, \\ u(x, 0) = U_1 (1 - l^{-1} x) + U_2 l^{-1} x, \quad u_t(x, 0) = V, \\ \text{а) } A = 2, B = 1, U_1 = 1, U_2 = 0, \\ \text{б) } A = 1, B = 2, U_1 = 0, U_2 = 1, \\ \text{в) } A = 1, B = 0. \end{aligned}$$

### Evaluation criteria of the RGR:

Credited if 2 tasks are solved correctly,

Not counted if 2 tasks are not solved.

### Homework assignments

In the 5th and 6th semesters, the student is presented with homework for each of the five modules. Each homework consists of 2 tasks. The problem is considered to be solved correctly if the student provides a detailed and complete solution to it.

#### Typical homework assignment 1 (module 1)

1. An infinite string is given a transverse initial velocity  $v' = \text{const}$  at the interval  $(-1,1)$ , at  $a = 1$ . Solve the problem of string vibrations.
2. A semi-infinite string with a rigidly fixed end is excited by an initial deviation other than zero on a segment  $(c,3c)$  having the shape of a polyline with vertices  $c, 2c, 3c$ . Draw the string profile for the time points  $t = cv/2a, v=2,4,6$ .

#### Evaluation criteria

Evaluation indicator	Distribution of points
Items 1 and 2 are not fulfilled	0
Items 1 or 2 have been completed	1
Maximum point	2

#### Typical homework 2 (module 2)

1. Find the potential of a simple layer distributed with constant density on a sphere of radius  $R$ .
2. Calculate the volume potential for a ball of radius  $R$ , with a given constant density.

#### Evaluation criteria

Evaluation indicator	Distribution of points
Items 1 and 2 are not fulfilled	0
Items 1 or 2 have been completed	2
Maximum point	4

### Typical homework 3 (module 3)

1. Solve the problem of free oscillations of a homogeneous circular membrane of radius  $R$ , fixed along the edge, if the initial deviation has the form of a paraboloid of rotation, and the initial velocity is zero.
2. Solve the problem of free oscillations of a homogeneous circular membrane of radius  $R$ , fixed along the edge, if the initial deviation has the form  $f(r)$ , and the initial velocity changes according to the law  $F(r)$ .

#### Evaluation criteria

Evaluation indicator	Distribution of points
Items 1 and 2 are not fulfilled	0
Items 1 or 2 have been completed	2
Maximum point	4

### Typical homework 4 (module 1)

1. Find the generalized derivative of the function  $f(x) = \theta(x) \cdot \cos x$ .
2. Find the generalized derivative of the function  $f(x) = |x| \sin x$ .

#### Evaluation criteria

Evaluation indicator	Distribution of points
Items 1 and 2 are not fulfilled	0
Items 1 or 2 have been completed	2
Maximum point	5

### Typical homework 5 (module 2)

1. Find the generalized derivative  $\theta(-x)$ .
2. Find the generalized derivative  $(\text{sign } x)$ .

#### Evaluation criteria

Evaluation indicator	Distribution of points
Items 1 and 2 are not fulfilled	0
Items 1 or 2 have been completed	2
Maximum point	5

### Tasks for classroom work

In the 5th and 6th semesters, the student is presented with classroom work on each of the five modules. Each classroom work consists of two tasks. The problem is considered to be solved correctly if the student provides a detailed and complete solution to it.

### Typical classroom assignment 1 (module 1)

1. Solve the problem of heat propagation in a thin homogeneous rod of length  $l$  ( $0 < x < l$ ), the side surface of which is thermally insulated, and the ends  $x = 0$ ,  $x = l$  are maintained at zero temperature.
2. A thin homogeneous rod of length  $l$  ( $0 < x < l$ ) is given, the side surface of which is thermally insulated. Find the temperature distribution  $u(x,t)$  in the rod if the ends of the rod are thermally insulated.

#### Evaluation criteria

Evaluation indicator	Distribution of points
M 1 and 2 are not fulfilled	0
Items 1 or 2 have been completed	1
Maximum point	3

### Typical classroom assignment 2 (module 2)

1. A homogeneous ball of radius  $R$  with the center at the origin is given. Determine the temperature inside the ball if its outer surface is maintained at zero temperature, and the initial temperature depends only on the distance from the center of the ball.
2. A thin homogeneous rectangular plate is given, the contour of which is maintained at zero temperature. The plate is fixed along the contour, the initial temperature distribution is set and it is required to determine the temperature of the plate at any time  $t > 0$ , assuming that there is no thermal exchange between the side surface of the plate and the environment.

#### Критерии оценки

Evaluation indicator	Distribution of points
M 1 and 2 are not fulfilled	0
Items 1 or 2 have been completed	3
Maximum point	6

### Typical classroom assignment 3 (module 3)

1. Solve the Dirichlet problem for the Laplace equation in a circle of radius  $r=1$ , provided that  $u(1,t)=\cos\varphi$ .
2. Solve the Dirichlet problem for the Laplace equation in a circular ring ( $1 < r < 2$ ), provided that  $u(1,t)=v^0=\text{const}$ ,  $u(2,t)=v^2=\text{const}$ .

#### Evaluation criteria

Evaluation indicator	Distribution of points
Items 1 and 2 are not fulfilled	0
Items 1 or 2 have been completed	3
Maximum point	6

### Typical classroom assignment 4 (module 1)

1. Find the stationary temperature distribution  $u(x,y)$  in a rectangular homogeneous plate  $0 < x < a$ ,  $0 < y < b$  if its sides  $x=a$ ,  $y=b$  are covered with thermal insulation, and the other two sides  $x=0$ ,  $y=0$  are maintained at zero temperature and in the plate heat is released with a constant density  $q$ .
2. Find the distribution of the potential of the electrostatic field  $u(x,y)$  inside a rectangular box, two opposite faces ( $x=-a$ ,  $x=a$ ) of which have a potential  $v^\circ = \text{const}$ , and the remaining faces ( $y=-b$ ,  $y=b$ ) are grounded.

#### Evaluation criteria

Evaluation indicator	Distribution of points
Items 1 and 2 are not fulfilled	0
Items 1 or 2 have been completed	5
Maximum point	10

### Typical classroom assignment 5 (module 2)

1. Find the logarithmic potential for a circle of radius  $R$ , with density  $v^\circ = \text{const}$ .
2. Find the logarithmic potential of the double layer for the segment  $-a < x < a$ ,  $y=0$ , with a constant density  $v^\circ = \text{const}$ .

#### Evaluation criteria

Evaluation indicator	Distribution of points
Items 1 and 2 are not fulfilled	0
Items 1 or 2 have been completed	5
Maximum point	10

## Educational, methodological and informational support of the discipline

### 5.1. The list of basic and additional educational literature necessary for the development of the discipline

#### Basic literature:

1. Differential and integral equations. Variational calculus in examples and problems [Electronic resource] / Vasilyeva A. B. — M. : Fizmatlit, 2005.— 214s. — () .— Access to the text of the electronic edition is possible through the Electronic library system "University Library online". — ISBN 5-9221-0628-7 .— <URL:<http://www.biblioclub.ru/book/68123/>>.
2. Emelyanov, V.M. Equations of mathematical physics. Workshop on problem solving / V. M. Yemelyanov, E. A. Rybakina. – St. Petersburg: Lan, 2008. Access point: <http://e.lanbook.com/view/book/140/>
3. Equations of mathematical physics [Electronic resource] / Vladimirov V. S. — M. : Fizmatlit, 2008 .- 201 p. — Access to the text of the electronic edition is possible through the Electronic Library system "University Library online.— ISBN 978-5-9221-0310-7 .— <URL:<http://www.biblioclub.ru/book/68126/>>.

#### Additional literature:

4. Andreev, Alexander Nikolaevich. Selected chapters of the theory of differential equations [Text] : textbook / A. N. Andreev ; Kemerovo State University. - Kemerovo : [B. I.], 2012. - 111 p. [http://e.lanbook.com/books/element.php?pl1\\_cid=25&pl1\\_id=44307](http://e.lanbook.com/books/element.php?pl1_cid=25&pl1_id=44307)

### 5.2. The list of resources of the information and telecommunication network "Internet" and software necessary for the development of the discipline

#### The list of resources of the information and telecommunication network "Internet" and software necessary for the development of the discipline

1	Electronic library system "EB BASHGU"	Own electronic library of educational and scientific electronic publications, which includes publications of teachers of BASHGU	Authorized password access from anywhere on the Internet	Registration in the Library of BASHGU, further access from anywhere on the Internet	<a href="https://elib.bashedu.ru/">https://elib.bashedu.ru/</a>
2	Electronic library system "University Library online"	Full-text database of educational and scientific electronic publications	Authorized password access from anywhere on the Internet	Registration from the BASHGU network, further access from anywhere on the Internet	<a href="http://www.biblioclub.ru/">http://www.biblioclub.ru/</a>
3	Electronic library system of the	Full-text database of educational and	Authorized password	Registration from the BASHGU	<a href="http://e.lanbook.com/">http://e.lanbook.com/</a>

publishing house "Lan"	scientific electronic publications	access from anywhere on the Internet	network, further access from anywhere on the Internet	
all-Russian mathematical portal				<a href="http://www.mathnet.ru">www.mathnet.ru</a>
Electronic catalog of the National Bank of KemGU				<a href="http://www.library.kemsu.ru">www.library.kemsu.ru</a>
Scientific Electronic Library				<a href="http://www.elibrary.ru">www.elibrary.ru</a>
Electronic Library of the Faculty of Mechanics and Mathematics of Moscow State University				<a href="http://www.lib.mexmat.ru">www.lib.mexmat.ru</a>
New Electronic library				<a href="http://www.newlibrary.ru">www.newlibrary.ru</a>
Federal Portal of Russian Education				<a href="http://www.edu.ru">www.edu.ru</a>

## 6. The material and technical base necessary for the implementation of the educational process in the discipline

Name of special* premises and premises for independent work	Equipment of special rooms and rooms for independent work	List of licensed software. Details of the supporting document
<p>1. classroom for lecture-type classes: classrooms No. 501, 530 (physical education building - educational).</p> <p>2. classroom for seminar-type classes: classrooms No. 531,526 (physical education building - educational).</p> <p>3. classroom for group and individual consultations: classrooms No. 501, 530,526</p> <p>4. classroom for current control and intermediate certification: classrooms No. 531, 526 (physical education building - educational).</p> <p>5. premises for independent work: reading room No. 2 (physics and mathematics building - educational).</p>	<p><b>Audience № 501</b> Educational furniture, blackboard, personal computer. and system unit /Core i5-4460(3.2)/CIGABAYTE GV-N710D3-1GL/4Gb, Logitech Wireless Presenter R400 (210134000003592), Sony VPL-DX270 projector, manual ViewScreen Lotus 244x183 WLO-4304</p> <p><b>Audience № 530</b> Educational furniture, chalkboard wall</p> <p><b>Audience № 531</b> Educational furniture, chalkboard wall, multimedia projector Sony VPL-EX120, XGA, 2600 ANSI, 3.2 kg, ceiling mount for projector (2101068302), auditor board. YES32.</p> <p><b>Auditorium No. 526</b> Educational furniture, chalkboard wall Reading Room No. 2 Educational furniture,</p>	<p>1. Windows 8 Russian. Windows Professional 8 Russian Upgrade. Contract No. 104 dated 17.06.2013. Perpetual licenses. 2. Microsoft Office Standard 2013 Russian. Contract No. 114 dated 12.11.2014. Perpetual licenses.</p>

	educational and visual aids, fire safety stand, stationary monoblocks - 8 pcs, printer – 1 pc., scanner – 1 pc.	
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MINISTRY OF EDUCATION AND SCIENCE OF RUSSIA BASHKIR STATE UNIVERSITY  
FACULTY OF MATHEMATICS AND INFORMATION TECHNOLOGIES

**THE CONTENT OF THE WORK PROGRAM**

of discipline “ Partial differential equations” for the 5th semester

*(name of discipline)*

*full -time education*

form of training

<b>Type of work</b>	<b>The scope of the discipline</b>
The total labor intensity of the discipline (ZET / hours)	2/216
Study hours for contact work with the teacher:	
lectures	36
practical / seminar laboratory	
laboratory	36
others (group, individual consultation and other types of educational activities involving the work of students with the teacher) (FCR)	0,2
Study hours for independent work of students (SR)	35,8
Study hours for exam preparation / test /differentiated credit (Control)	

Control form(s):

exam \_\_\_\_\_ semester

test\_5\_ semester



**THE CONTENT OF THE WORK PROGRAM**

of discipline “ Partial differential equations” for the 5th semester

*(name of discipline)*

*full -time education*

form of training

<b>Type of work</b>	<b>The scope of the discipline</b>
The total labor intensity of the discipline (ZET / hours)	4/216
Study hours for contact work with the teacher:	121,9
lectures	16
practical / seminar laboratory	32
laboratory	
others (group, individual consultation and other types of educational activities involving the work of students with the teacher) (FCR)	1,7
Study hours for independent work of students (SR)	86,3
Study hours for exam preparation / test /differentiated credit (Control)	43,8

Control form(s):

Exam 6 semester

test \_\_\_\_\_ semester

№ n/a	Theme and content	The form of studying materials: lectures, practical classes, seminars, laboratory work, independent work and labor intensity (in hours)				Basic and additional literature recommended to students (numbers from the list)	Tasks for independent work of students	The form of ongoing monitoring of academic performance (colloquiums, control papers, computer tests, etc.)
		ЛК	ПП/СЕМ	ЛР	СР			
1	2	3	4	5	6	7	8	9
1.	Introduction	6	6	6	24,46	1 – 4	2.2, 2.3., 2.4, 2.11	Control work, RGR, classroom work
2.	Hyperbolic type equations	12	6	8	24,46	1 – 4	12.36 12.37, 12.38, 20.1, 20.2, 20.6, 20.9, 20.14, 20.15, 20.16	Control work, RGR, classroom work
3.	Hyperbolic type equations	14	8	8	24,46	1 – 4	13.5, 13.6, 13.7, 20.40, 20.41, 20.45	Control work, RGR, classroom work
4.	Elliptic type equations	12	6	8	24,46	1 – 4	16.1, 16.2, 16.3, 16.4, 16.5, 16.6, 16.7, 16.8, 16.9, 16.10, 16.13, 16.20	Control work, RGR, classroom work
5	Variable separation method (Fourier method)	8	6	6	24,46	1 – 4	16.21, 16.22, 16.24, 16.25, 16.26, 20.46, 20.18	Control work, RGR, classroom work
	<b>Total hours:</b>	52	32	36	122,3			

**Rating – discipline plan**

Partial differential equations

(the name of the discipline according to the working curriculum)

direction/specialty 01.03.02 Applied mathematics and Computer Science

course 3, semester 5

Types of educational activities of students	Score for a specific task	Number of assignments per semester	Points	
			Minimum	Maximum
<b>Module 1.</b>				
<b>Current control</b>			<b>0</b>	<b>10</b>
1. Classroom work, seminar work				<b>3</b>
2. Homework				<b>2</b>
<b>Border control</b>				
Control work	<b>1</b>	<b>5</b>		<b>5</b>
<b>Module 2.</b>				
<b>Current control</b>				<b>20</b>
1. Classroom work, seminar work				<b>6</b>
2. Homework				<b>4</b>
<b>Border control</b>				
Control work	<b>2</b>	<b>5</b>		<b>10</b>
<b>Module 3.</b>				
<b>Current control</b>				<b>20</b>
1. Classroom work, seminar work				<b>6</b>
2. Homework				<b>4</b>
<b>Border control</b>				
Control work	<b>2</b>	<b>5</b>		<b>10</b>
1. Student Olympiad or essay contest			<b>0</b>	<b>5</b>
2. Volunteer work during Olympiads and conferences			<b>0</b>	<b>5</b>
<b>Attendance (points are deducted from the total amount of points scored)</b>				
1. Attending lectures			<b>0</b>	<b>-6</b>
2. Attending practical (seminar, laboratory classes)			<b>0</b>	<b>-10</b>
<b>Final control</b>				
<b>Test</b>			<b>0</b>	<b>60</b>
<b>Total</b>			<b>60</b>	<b>100</b>

## Rating – discipline plan

### Partial differential equations

(the name of the discipline according to the working curriculum)

direction/specialty 01.03.02 Applied mathematics and Computer Science

course 3, semester 6

Types of educational activities of students	Score for a specific task	Number of assignments per semester	Points	
			Minimum	Maximum
<b>Module 1.</b>				
<b>Current control</b>			<b>0</b>	<b>35</b>
1. Classroom work, seminar work				<b>10</b>
2. Homework				<b>5</b>
<b>Border control</b>				
Control work	<b>10</b>	<b>2</b>		<b>20</b>
<b>Module 2.</b>				
<b>Current control</b>				<b>20</b>
1. Classroom work, seminar work				<b>6</b>
2. Homework				<b>4</b>
<b>Border control</b>				
Control work	<b>2</b>	<b>5</b>		<b>10</b>
<b>Module 3.</b>				
<b>Current control</b>				<b>35</b>
1. Classroom work, seminar work				<b>10</b>
2. Homework				<b>5</b>
<b>Border control</b>				
Control work	<b>4</b>	<b>5</b>		<b>20</b>
1. Student Olympiad or essay contest			<b>0</b>	<b>5</b>
2. Volunteer work during Olympiads and conferences			<b>0</b>	<b>5</b>
<b>Attendance (points are deducted from the total amount of points scored)</b>				
1. Attending lectures			<b>0</b>	<b>-6</b>
2. Attending practical (seminar, laboratory classes)			<b>0</b>	<b>-10</b>
<b>Final control</b>				
<b>Exam</b>			<b>0</b>	<b>30</b>
<b>Total</b>			<b>45</b>	<b>100</b>