RUSSIAN MINISTRY OF SCIENCE AND EDUCATION

FEDERAL STATE BUDGETARY EDUCATIONAL INSTITUTION OF HIGHER EDUCATION «BASHKIR STATE UNIVERSITY»

FACULTY OF MATHEMATICS AND INFORMATION TECHNOLOGIES

Approved: at the department meeting <u>Protocol # 5 from February 28, 2022</u> Head of the department Coordinated with: EMC chairman of the faculty/institute



Z. Yu. Fazullin

A.M. Efimov

WORKING PROGRAM OF DISCIPLINE (MODULE)

Discipline Analysis I

(name of the discipline)

Obligatory part

(name of the part enclosing the discipline (obligatory, formed by participants of the educational activity, facultative))

bachelor (undergraduate) program

Course of training (speciality) 01.03.02 Applied mathematics and informatics (code and name of the course of training (speciality))

Subdivision of the course of training (profile) Applied programming and data analysis

(name of the profile of training)

Qualification (level of training) <u>bachel</u>or

(name of the level of training)

Designer (compiler):		
associate professor of the MA		
department, PhD	Man	
	Sunder	A.A. Makhota
		'

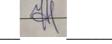
For enrollment of: 2022

Ufa 2022

Designer: <u>associate professor, PhD Alla Aleksandrovna Makhota</u>

The working program of the discipline is approved at the meeting of the department of Mathematical Analysis, Protocol # 5 from February 28, 2022

Head of the department



Z. Yu. Fazullin

The addenda and updates introduced into the working program of the discipline are approved at the meeting of the department of Higher algebra and geometry, protocol # 11 from June , «10 » 2022.

Head of the department

Z. Yu. Fazullin

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1. The list of planned learning outcomes of the discipline, correlated with the planned results of the educational program

Category (group) of competencies (if there is exist a GPC)	Competence to be formed (with the code)	Code and name of the indicator of competence achievement	Learning outcomes for the discipline
Theoretical and practical basics of professional activity	GPC-1. Able to apply the fundamental knowledge obtained in the mathematical and/or natural sciences and use them in professional activities	GPC-1.1. Possesses basic knowledge obtained in the field of mathematical analysis.	Know the basic concepts of disciplines, basic theorems and consequences, methods of solution and analysis of typical problems
		GPC-1.2. Is able to use them in professional activities.	Be able to use in practice the knowledge of disciplines, correctly formulate tasks and reasonably choose methods of their solutions
		GPC-1.3. Has the skills to choose the methods of solving the problems of professional based on theoretical knowledge.	Master the basic mathematical knowledge and its application to the solution of problems theoretical and applied character

Mastering the discipline must lead to forming of the following competence (GPC-1):

2. The discipline "Mathematical Analysis" is part of the basic part. The discipline is studied at the 1.2 courses in semesters 1-3.

The purpose of the discipline: studying the basics of mathematical analysis, combining the theory of real numbers, the theory of limits, the theory of series, differential and integral calculus and their direct applications; development of the ability to understand and apply in the research and applied activities of the modern mathematical apparatus; development of logical, heuristic and algorithmic thinking.

To master the discipline, competencies formed in the study of school mathematics course are necessary. The discipline is closely related to such disciplines as "Complex analysis", "Applied Functional Analysis", "Differential Geometry", "Differential Equations", "Probability Theory and Mathematical Statistics", "Optimization Methods", "Equations with Partial Derivatives".

3. The content of the working program (the volume of the discipline, types and types of classes, educational and methodological support of independent work of students)

The content of the working program is presented in Appendix \mathbb{N}_{2} 1.

4. Evaluation funds for the discipline

4.1. List of competencies and indicators of achievement of competencies with the planned learning outcomes related to them in the discipline. Description of criteria and scales for evaluating the results of training in the discipline.

Code and definition of each competence.

GPC-1: Ability of applying the fundamental knowledge gained in the field of mathematical and (or) natural sciences and using them in professional activities.

	Planned	Evaluation criteria for the results						
Level of competence acquirement	learning outcomes (indicators of achievement of a predetermine d level of acquired competencies)	2 (" Non- satisfactory")	3 ("Satisfactory")	4 ("Good")	5 ("Excellent")			
First level	To know: basic concepts of disciplines, basic theorems and their consequences, methods of solving and analyzing typical problems	Lack of knowledge	Incomplete ideas about the basic concepts of the discipline, the main theorems and their consequences, methods of solving and analyzing typical problems	Formed knowledge and skills, but containing separate gaps in the basic concepts of the discipline, the main theorems and their consequences, methods of solving and analyzing typical problems	Formed systematic knowledge and skills on the basic concepts of the discipline, the main theorems and their consequences, methods of solving and analyzing typical problems			

Second level	Be able to: use the knowledge of the disciplines in practice, correctly formulate tasks and reasonably choose methods of their solution	Lack of skills	Partial skills of using the knowledge of the disciplines in practice, correctly formulate tasks and reasonably choose methods of their solution	Generally formed skills, but containing some gaps in the ability of using knowledge of the disciplines in practice, correctly formulate tasks and reasonably choose methods of their solution	Well-formed ability using the knowledge of the disciplines in practice, correctly formulate specific tasks and reasonably choose methods of their solution
Third level	Be in possession of: basic mathematical knowledge and its application for solving theoretical and applied problems	Lack of formed skills	Generally successful, but non-systematic application of basic mathematical knowledge for solving theoretical and applied problems	Generally successful, but containing some gaps in the application of basic mathematical knowledge for solving theoretical and applied problems	Successful acquisition of basic mathematic al knowledge and its application for solving theoretical and applied problems

Level of competence	Planned learning outcomes	Evaluation criteria for the results (pass/fail rating)				
acquirement	(indicators of achievement of a predetermined level of acquired competencies)	Grade of "non pass" ("failed")	Grade of "pass" ("passed")			
First level	To know: basic concepts of the disciplines, basic theorems and their consequences, methods of solving and analyzing typical problems	Incomplete ideas about the basic concepts of the discipline, the main theorems and their consequences, methods of solving and analyzing typical problems	Well-formed systematic ideas about the main concepts of the discipline, the main theorems and their consequences, methods of solving and analyzing typical problems			
Second level	knowledge of the disciplines in practice, correctly formulate tasks and	Partial skills of using the knowledge of the disciplines in practice, correctly formulate tasks and reasonably choose methods of their solution	Well-formed ability of using the knowledge of the disciplines in practice, correctly formulate tasks and reasonably choose methods of their solution			
Third level	Be in possession of: basic mathematical knowledge and its application for solving theoretical and applied problems	Generally successful, but non systematic application of basic mathematical knowledge for solving theoretical and applied problems	Successful acquirement of basic mathematical knowledge and its application for solving theoretical and applied problems			

Evaluation criteria are the points which are given by the teacher to each student for different types of activities (evaluation tools) at the end of the study of each module (section of a discipline) that are listed in the rating plan of the discipline (for the exam: current control -40 points maximum; periodical control -30 points maximum, bonus points -10 points maximum. For the grade of "pass": current control -50 points maximum; periodical control -50 points maximum; periodical control -10 points maximum.

Grading scale:

(for the exam:

45 to 59 points - "satisfactory";

<u>60 to 79 points</u> - "good";

80 points – "excellent".

for the grade of "pass":

"passed" - from 60 to 110 rating points (including 10 bonus points),

"failed" - from 0 to 59 rating points).

4.2. Standard control tasks or other materials necessary for evaluating the results of training in the discipline, correlated with

Indicator of the achieved competence	Results of training in the discipline	Evaluation tools		
GPC-1. 1. Possesses basic knowledge gained in the field of mathematical analysis	To know: the basic concepts of the discipline, the main theorems and their consequences, methods of solving and analyzing typical problems			
GPC-1. 2. Knows how to use them in professional activities	Be able to: use the knowledge of the discipline in practice, correctly formulate tasks and reasonably choose methods of their solution	Laboratory work, test work, pass/fail rating		
GPC-1.3. Possesses the skills to choose methods of solving problems in professional activity based on theoretical knowledge.	To possess: basic mathematical knowledge and its application for solving theoretical and applied problems	Laboratory work, test work, pass/fail rating		

the indicators of competence achievement established in the educational program. Methodological materials defining the procedures for evaluating the results of training in the discipline

Examination tickets

Exam and credit is an assessment tool for all stages of mastering the competencies.

Questions for the exam (first semester):

- 1. Numerical sets. The method of mathematical induction. Actions on rational numbers. Properties. Irrationality.
- 2. Scheme for constructing a set of real numbers. Infinite decimal fractions (idf).
- 3. Comparison of idf. Approximation of real numbers by rational numbers.
- 4. Facets of number sets. Principles of upper and lower edges of a number set.
- 5. The limit of a sequence. Properties of the limit.
- 6. Arithmetical operations on sequences.
- 7. Limits of a monotone sequence.
- 8. The lemma of imbedding segments.
- 9. Lemma on finite coverage.
- 10. The concept of a subsequence. The lemma for distinguishing a convergent subsequence.
- 11. The notion of a function. Elementary functions.
- 12. Two definitions of limit functions in the language of " δ - ϵ " and in the language of sequences

(reduction to the case of a sequence).

- 13. The first remarkable limit.
- 14. The second remarkable limit.
- 15. Arithmetic operations on limits. Limit transition in inequalities.
- 16. Limit of a monotone function.
- 17. Comparison of functions. O-symbols. Criterion for equivalence of functions.
- 18. Continuity of function. 3 definitions. Classification of points of discontinuity. Examples. Arithmetic properties of continuity.
- 19. Continuity of elementary functions.
- 20. Properties of continuous functions (Theorems: about constancy of sign, Cauchy about passing through zero, Cauchy about intermediate value).
- 21. Weierstrass's first and second theorems.
- 22. Reversibility of a continuous monotone function.
- 23. Continuity of a complex function.
- 24. Uniform continuity. Cantor's theorem.
- 25.Definition of the derivative of a function. The geometrical meaning of the derivative. Examples of the calculation of derivatives. Table of derivatives of basic elementary functions. Representation for the increment of a function. Unilateral derivatives.
- 26. The derivative of an inverse function.
- 27. Rules for calculating derivatives (derivative of the sum, product, and quotient of two functions).
- 28. The derivative of a composite function.
- 29.Definition of a differentiable function. The relationship between differentiability and the existence of a derivative function. The differential of a function.
- 30. Invariance of the form of the first differential.
- 31.Basic theorems of differential calculus. (Fermat's Lemma, Rolle's Theorem, Lagrange's Theorem, Cauchy's Theorem). Geometrical interpretation of these theorems.
- 32. Derivatives and higher order differentials.
- 33. Taylor formula for a polynomial. Calculation of coefficients.
- 34.Decomposition of an arbitrary function by Taylor's formula. The residual term in the Peano form.
- 35.Lopital's rule. Uncertainty $\left(\frac{0}{0}\right)$, $\left(\frac{\infty}{\infty}\right)$
- 36. The condition of monotonicity of a function in a given interval. Extremes of a function. Necessary and sufficient conditions for the presence of an extremum.
- 37. Convex functions. Geometrical interpretation. Conditions for convexity of a function.
- 38. Points of inflection. Asymptotes. Scheme for plotting the graph of a function by characteristic points.

1. The concept of the antiderivative function. Properties of the indefinite integral. Integration by the method of replacement of a variable.

2. Indefinite integral. Integration by parts. Integration of expressions containing trigonometric functions.

- 3. An indefinite integral. Simple fractions and their integration.
- 4. Decomposition of regular fractions into partial fractions.
- 5. Integration of expressions of the form $R(x)^m \sqrt{\frac{ax+\beta}{\gamma x+\delta}}$
- 6. Integrating a binomial differential.
- 7. The area problem.
- 8. Definition of a definite integral. A necessary condition for integrability.
- 9. Definitions of Darboux sums. Their properties.
- 10. The condition for the existence of a definite integral.
- 11. Classes of integrable functions (continuous, monotone and bounded).
- 12. Properties of integrable functions.
- 13. Properties of a definite integral expressed by equations.
- 14. Properties of the definite integral expressed by inequalities.
- 15. The average value theorem. The generalized mean value theorem.
- 16. The definite integral as a function of the upper limit.
- 17. The Newton-Leibniz formula.
- 18. The concept of a curve. The arc length of a curve. Straightenable curves.
- Definition of the concept of area of a plane figure. The property of additivity. Area as a limit.
- 20. The class of quadratic figures. Expression of area by an integral.
- 21. Area of a curvilinear sector.
- 22. Definition of the concept of volume. Expression of volume by an integral.
- 23. The area of a surface of rotation.
- 24.Space \mathbb{R}^n . Metric. Norm of an element.
- 25.0pen and closed sets in \mathbb{R}^n . Closedness criterion. Compact in \mathbb{R}^n .
- 26. The limit of a function of many variables. Reduction to the case of a sequence. The Bolzano-
- Weirstrass theorem for the extraction of a convergent subsequence.
- 27.Repeated limits. The relation between double and repeated limits.
- 28.Definition of the continuity of a function of many variables. Basic properties of continuous functions.
- 29. Partial derivatives and partial differentials. The total increment of a function.
- 30. The total differential of a function of many variables.

31. The derivatives of a composite function of many variables.

32. The derivative in a given direction. Gradient.

33. The derivatives of higher orders of a function of many variables. Theorem on mixed derivatives.

34.Extremes of functions of many variables. A necessary condition.

35.Extremes of functions of many variables. A sufficient condition.

36. The concept of numerical series. The sum of an infinite geometrical progression. Prerequisite for the convergence of a series. Cauchy criterion.

37.Significant positive series. Sufficient conditions for convergence (boundedness, comparison sign, Dalembert's test).

38. Famously positive series. Sufficient conditions for convergence (boundedness, Cauchy's test, Raabe's test, integral test).

39. Absolute and conditional convergence of numerical series. Leibniz's test of the convergence of a sign-convergent series.

40. Arbitrary numerical series. The Abel transformation. Abel's and Dirichlet's tests.

Structure of the examination ticket:

1. Theoretical question.

2. Theoretical question

Grading criteria (in points):

- 25-30 points if student demonstrates the knowledge of 80% or more of the required educational material in the discipline.

- $\underline{17-24}$ points if student demonstrates the knowledge from 60% to 79% of the required educational material in the discipline.

<u>- 10-16 points</u> if student demonstrates the knowledge from 45% to 59% of the required educational material in the discipline.

- <u>1-10</u> points if student demonstrates the knowledge less than 45% of the required educational material in the discipline.

An example of examination tickets is given in Appendix 2.

Sample questions for verbal examination (Semester 1)

Module 1.

- 1. Irrationality.
- 2. Infinite decimals.
- 3. The method of mathematical induction.
- 4. Properties of natural numbers.
- 5. Properties of integers.
- 6. Properties of rational numbers.
- 7. The principle of the upper (lower) bound of a number set.
- 8. The limit of a sequence.

Module 2.

- 1. The limit of a function.
- 2. The first remarkable limit.
- 3. The second remarkable limit.
- 4. The number e.
- 5. The limit transition in equality and inequality.
- 6. Continuity of a function at a point.
- 7. Continuity of elementary functions.

Module 3.

- 1.. Properties of continuous functions.
- 2.. Uniform Continuity.
- 3. The derivative of a function of one variable. Geometrical meaning.
- 4. Derivatives of elementary functions.
- 5. The derivative of the sum, product, and quotient of two functions.
- 6. The derivative of a complex function.

Module 4.

- 1. The invariance of the form of the first differential.
- 2. Basic theorems of differential calculus.
- 3. Taylor's formula.
- 4. Lopital's rule.
- 5. Extremes of functions of one variable.

Sample questions for verbal examination (Semester 2)

Module 1.

- 1. Definition of the antiderivative function.
- 2. Properties of an indefinite integral.
- 3. Substitution of a variable in an indefinite integral.
- 4. Integration by parts.

Module 2.

- 1. The definite integral.
- 2. The necessary condition for integrability.
- 3. The upper and lower sums of Darboux.
- 4. The condition for the existence of a definite integral.
- 5. The average value theorem.

- 6. The Newton-Leibniz formula.
- 7. Calculation of arc length.
- 8. The area of a curvilinear sector.

Module 3.

- 1. A necessary condition for the convergence of a numerical series.
- 2. The Cauchy criterion.
- 3. Cauchy's test.
- 4. The test of comparison.
- 5. Leibniz's test.
- 6. Absolute and conditional convergence of numerical series.
- 7. Abel's and Dirichlet's tests.

Each student is given one question from each module during the semester.

Each question is graded 2 points. These points are taken into account when calculating the rating only for the exam.

Grading criteria (in points):

<u>- 15 points</u> are awarded to a student if he/she gives a clear, detailed and correct answer to the question;

<u>- 10-14 points</u> are awarded if the student gave a correct answer, but may be incomplete or did not answer 1-2 additional questions;

<u>- 1-9 points</u> are awarded if the student gave an incorrect answer to the question, but answered additional questions or gave a fragmented, but correct answer to the main question, but did not answer additional questions;

<u>- 0 points</u> will be assigned to the student if he or she fails to answer the main question and does not answer any additional questions.

Work in class

Class work: solving problems from the book [3]. Each activity is graded 1 point.

Assignments for laboratory work

Each lab work consists of several problems on the topic being tested. Each student is given a separate version. All laboratory works are performed by each student independently. The problem is considered solved correctly if the correct solution is written in detail and clearly. Upon completion of the laboratory work, the student submits a report that contains the solutions to the problems on the given laboratory work.

Examples of a variant of the laboratory work:

1. Proof
$$1^3 - 2^3 + 3^3 - 4^3 + \dots + (2n-1)^3 - (2n)^3 = -n^2(4n+3)$$
.
2. Find limits a) $\lim_{n \to \infty} \frac{(2n+1)^2 - (n+1)^2}{n^2 + n + 1}$; b) $\lim_{n \to \infty} \frac{n^3 \sqrt{3n^2} + \sqrt[4]{4n^8 + 1}}{(n + \sqrt{n})\sqrt{7 - n + n^2}}$; c) $\lim_{n \to \infty} \left(\frac{n+2}{n-1}\right)^n$;
d) $\lim_{n \to 0} \frac{n^3 - n^2 + 2n}{n^2 + n}$; e) $\lim_{n \to 3} \frac{n^2 + n - 12}{\sqrt{n - 2} - \sqrt{4 - n}}$.
3. Check if the sequence converges $x_n = \frac{5\sqrt{n^3} \cos n!}{n+2}$.

4.
$$x_n = -1 + 3(-1)^{n+1}$$
. Find $\inf x_n$, $\sup x_n$.

5.
$$x_n = (-1)^{n+1}n$$
. Find $\lim_{n \to \infty} x_n$, $\overline{\lim_{n \to \infty} x_n}$.

Description of the evaluation methodology for the tests:

Grading Criteria (in points): The maximum number of points a student can earn for a correctly solved problem is written in front of each problem. For solving the problem you can get 0.5, 1, 1.5, 2 or 3 points depending on the correctness of the solution given (there can be inaccuracies in the solution, incomplete solution, not completely correct solution).

Examples of a variant of the test:

1. Proof $\lim_{n \to \infty} \frac{2n^4 + 1}{2n^4 + n^3} = 1$ (3 points) 2. Define a sequence that converges to $+\infty$. (1 points)

3. Proof 1.2.3+2.3.4+3.4.5+...+ $n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$. (3 points)

Exemplary topics for course papers

Investigate the function $f(x) = \dots$

Algorithm

- 1. Find the area of definition. Identify singular points (points of discontinuity).
- 2. Check for vertical asymptotes at the points of discontinuity and on the boundaries of the definition.
- 3. Find the points of intersection with the axes of coordinates.
- 4. Determine whether the function is even or odd.
- 5. Determine if the function is periodic or not (only for trigonometric functions).
- 6. Find points of extremum and intervals of monotonicity.
- 7. Find inflection points and convex-concave intervals (if possible).
- 8. Investigate the behavior at infinity. Find inclined asymptotes.
- 9. Select additional points and calculate their coordinates.
- 10. Plot the graph and asymptotes.
- 11. Calculate the integral on the interval [a; b] using the trapezium method.
- 12. Check your results in any program (Maple, Alpha Wolfram etc.).

4.3. Rating – plan of the discipline

Rating-plan of the discipline is given in Appendix 3.

5. Educational, methodic and informational support of the discipline.

5.1. List of references to primary and complementary educational literature

necessary for acquiring the discipline.

Primary literature

1. Fichtenholz G.M. Course of differential and integral calculus. In 3 vols. M.: Fizmatlit, 2001. (in Russian) <u>http://biblioclub.ru/index.php?page=book_red&id=83037</u>

2. William F. Trench Introduction to Real Analysis <u>https://digitalcommons.trinity.edu/mono/7/</u>

3. Demidovich B.P. Collection of Problems and Exercises in Mathematical Analysis: Textbook for Universities Moscow: CheRo, 1997.

http://biblioclub.ru/index.php?page=book_red&id=459722

Auxiliary literature:

5. ?????????

5.2. List of the Internet resources and software necessary for acquiring the discipline, including professional data bases and reference systems.

- 1. Library of Bashkir State University http://lib.bashedu.ru
- 2. BashSU Electronic Library System <u>https://elib.bashedu.ru</u>
- 3. Universitary WebWork server: http://webwork-okko.bashedu.ru/webwork2/.

6. Hardware equipment, materials and rooms necessary for implementing the educational process in the discipline.

Names of specialized rooms, rooms and laboratories	Activity form	Name of the equipment/software
1	2	3
Rooms 501,517, 528 or any other room according to the current time table	Lectures	The board for writing, projector and screen
Rooms 517, 503 or any other room according to the current time table	Laboratory	The board for writing, projector and screen
Library, reading halls	Individual work	Internet. The universitary WebWork server

FEDERAL STATE BUDGETARY EDUCATIONAL INSTITUTION OF HIGHER EDUCATION «BASHKIR STATE UNIVERSITY»

CONTENT OF THE WORKING PROGRAM

of the discipline <u>Analysis I</u> for semester <u>1</u>

Activity	Duration
Total duration of the discipline (CUD / hours)	7/252
Academic hours for the work with instructor	
lectures	54
laboratory	72
other (consultation in group or individually and other forms of learning activities assuming collaboration of learners with instructor)	1,7
Academic hours for individual work of learners	71,5
Academic hours for preparing to exam/credit test/differentiated credit test (Grading)	52,8
laboratory	

Final grading:

exam in semester 1

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CONTENT OF THE WORKING PROGRAM

of the discipline <u>Analysis I</u> for semester 2

Activity	Duration
Total duration of the discipline (CUD / hours)	6/216
Academic hours for the work with instructor	
lectures	48
laboratory	64
other (consultation in group or individually and other forms of learning activities assuming collaboration of learners with instructor)	1,7
Academic hours for individual work of learners	58,5
Academic hours for preparing to exam/credit test/differentiated credit test (Grading)	43,8
laboratory	

Final grading:

exam in semester 2

	Semester 1										
Item no.	Topic and its content	Learning forms: lectures, seminars, laboratory, and individual work with duration (in academic hours)				Primary and auxiliary literature (numbers in the reference list)	Task for individual work of learners	Forms of current grading (clolloquia, quizes, computer tests etc)			
		LEC	SEM	LAB	IND						
1	2	3	4	5	6	7	8	9			
1.	The subject of mathematical analysis, information about sets and logical symbolism, mapping and functions.	2	-	2	4	1-3,	[3]: 22-29, 151-154	Theoretical survey			
2.	Real numbers: algebraic properties of the set R of real numbers. Infinite decimal fractions.	3	-	4	4	1-3	[3]: 3, 4, 6, 10,	Theoretical survey			
3.	The axiom of set completeness R. Basic principles of completeness of the set R: existence of exact upper (lower) bound of the numerical set, principle of nested segments, the lemma of finite coverage.	3	_	2	4	1-3	[3]: 16, 17, 20, 21	Laboratory work No.1			
4.	Limit theory: the limit of a numerical sequence; basic properties and signs of the existence of limit.	2	-	6	4	1-3	[3]: 48, 52, 57, 60, 63	Laboratory work No.1			

5.	Limit points of a set and Bolzano-Weierstrass theorem for the separation of a convergent subsequence; the limit of a monotone sequence.	3	-	2	4	1-3	[3]: 69,79, 101, 104-109	Laboratory work No.1
6.	Number "e", upper and lower limits; Cauchy criterion for the existence of limit.	3	-	2	4	1-3	[3]: 84, 85, 74, 72, 111-114	Theoretical survey
7.	The limit of a function at a point. Various definitions and their equivalences. The Cauchy criterion for the existence limit.	3	-	6	4	1-3	[3]: 406, 407	Laboratory work No.2
8.	Properties of limits. Arithmetic operations on limits. The 1st and 2nd remarkable limits.	3	-	4	4	1-3	[3]:70,72,74,76,145 -147	Laboratory work No.2
9.	Classification of infinitesimal and infinitely large. Symbols "O", "o".	4	-	4	4	1-3	[3]: 647-651	Laboratory work No.2
10.	Continuous functions: local properties of Continuous functions; Continuity of a function from a function; point of discontinuity; Boundedness of a function that is continuous on a segment.	4	-	6	4	1-3	[3]: 674-728	Laboratory work No.2, Theoretical survey
11.	Existence of maximum and minimum values; Uniform continuity of a function that is continuous on the segment.	4	-	4	4	1-3	[3]: 794-801	Laboratory work No.2
12.	Monotone functions, existence and continuity of inverse function, continuity of elementary functions.	4	-	4	4,8	1-3	[3]: 767-772	Theoretical survey

13.	Differentials and Derivatives: The differentiability of a function at a point; the derivative at a point, the differential and their geometric meaning; the mechanical meaning of the derivative; the rules of differentiation; derivatives and differentials of higher orders; the Leibniz's formula.	4	-	6	5	1-3	[3]: 828, 845-859, 888-892,920-932, 961-966, 1039- 1042, 1055-1063, 1085-1090, 1099, 1134-1141	Laboratory work No.3
14.	Basic theorems of differential Rolle's, Lagrange's and Cauchy's theorems on finite increments. Basic theorems of differential Local Taylor formula; asymptotic expansions of elementary functions; Taylor formula with residual term.	4	-	6	5	1-3	[3]: 1251, 1377- 1387	Theoretical survey
15.	Application Differential The application of differential calculus to the study of functions. Signs of constancy, monotonicity, extremes, convexity, inflection points.	4	_	10	8	1-3	[3]: 1479-1498	Laboratory work No.3
16.	Lopital's rule	4		4	5	1-3	[3]: 1345-1364	Laboratory work No.3

					Semester 2			
Item no.	Topic and its content		rms: lectures, s ndividual work (in academ	with duratio		Primary and auxiliary literature (numbers in the reference list)	Task for individual work of learners	Forms of current grading (clolloquia, quizes, computer tests etc)
		LEC	SEM	LAB	IND			
1	2	3	4	5	6	7	8	9
	The original. The indefinite integral and its properties. Table of formulas for integration. Substitution of a variable. Integration by parts.	6		8	8	1-3	[3]: 1628-1632, 1643-1660	Theoretical survey
2.	Integration of rational functions and Irrational functions.	6		8	8	1-3	[3]: 1870-1887, 1929-1935, 1939-1947, 1955, 1957	Verbal questioning
3.	The definite integral: Problems leading to the concept of the definite integral. The definite integral Riemann; the criterion of integrability; integrability of a continuous function, monotone function and bounded function with a finite number of points	6	_	8	8	1-3	[3]: 2210-2213, 2220- 2223, 2242-2248	Laboratory work No.1, verbal questioning

4.	Properties of the definite integral. The average value theorem. Differentiation by a variable upper limit. The existence of a first form from a continuous of a function. Relation of the definite integral to the indefinite: Nyton-Leibniz formula. Substitution of a variable; integration by parts; arc length and other Geometrical, mechanical and physical applications.	6	_	8	8	1-3	[3]: 2398-2406, 2413- 2415,2418- 2421,2436,2440,2447, 2474 -2476,2493-2498	Laboratory work No.1, verbal questioning
5.	Functions of Many Variables: Euclidean Space. An overview of the basic metric and topological characteristics of point sets of Euclidean space. Limits, continuity of a function of many variables.	8	-	10	10	1-3	[3]: 3184-3192	Laboratory work No.3, verbal questioning
6.	Differential and partial derivatives of a function of many variables. Directional derivatives, gradient. Sufficient condition of differentiability; tangent and normal to surface.	6	_	8	8	1-3	[3]: 3214-3228, 3245, 3269-3271, 3283, 3284, 3293-3297	Laboratory work No.3

8.	Numerical series: convergence and sum of numerical series; Cauchy criterion; familiar- constant series; comparison of series; Cauchy's convergence; Cauchy's integral criterion of convergence. convergence.	5	-	8	4	1-3	[3]: 2548-2551, 2580-2582, 2626-2635	Laboratory work No.4
9.	Leibniz's test; absolute and conditional convergence; Abel transformation and its application to series; permutation of Absolute Convergent Series; Riemann's Theorem; operations on series; double series; concept of infinite products.	5	-	6	4,5	1-3	[3]: 2667-2670, 2675-2689	Laboratory work No.5, verbal questioning
	Course papers		-			1-3	A properly formatted study paper with elements of independent research on a proposed topic, accompanied by appropriate examples and graphs that illustrate the theoretical material studied.	
	Всего часов:	48		64	58,5			

Appendix 2

An example of examination tickets (semester 1)

FEDERAL STATE BUDGETARY EDUCATIONAL INSTITUTION OF HIGHER EDUCATION «BASHKIR STATE UNIVERSITY» FACULTY OF MATHEMATICS AND INFORMATION TECHNOLOGIES Department of Mathematical Analysis

EXAMINATION TICKET #1 on "Analysis I"

- 1. Numerical sets. The method of mathematical induction. Actions on rational numbers. Properties. Irrationality.
- 2. Lopital's rule. Uncertainty $\left(\frac{0}{0}\right)$, $\left(\frac{\infty}{\infty}\right)$

An example of examination tickets (semester 2)

FEDERAL STATE BUDGETARY EDUCATIONAL INSTITUTION OF HIGHER EDUCATION «BASHKIR STATE UNIVERSITY» FACULTY OF MATHEMATICS AND INFORMATION TECHNOLOGIES Department of Mathematical Analysis

EXAMINATION TICKET #2 on "Analysis I"

- 1. An indefinite integral. Simple fractions and their integration.
- 2. The derivative in a given direction. Gradient.

Rating – plan of the discipline

Analysis I______(the name of the discipline according to the working curriculum)

Direction 01.03.02 Applied Mathematics and Informatics Grade 1, semester 1

Rating-plan No. 1 (exam)

Points for a	Number	Po	ints
specific	of tasks	minimum	maximum
task	per		
Module 1. T	he limit of th	e sequence.	
		0	10
1	10	0	10
		0	7,5
2,5	3	0	7,5
. The limit of the	he function. (Continuity.	
		0	10
1	10	0	10
		0	7,5
2,5	3	0	7,5
e 3. Differentia	bility of the f	function.	
		0	10
1	10	0	10
		0	7,5
2,5	3	0	7,5
Iodule 4. Resea	arch of functi	ion	
		0	10
1	10	0	10
		0	7,5
2.5	2	0	
2,5	3	U	7,5
Lanus noints			
		0	5
+			
		0	5
		V	3
leducted from (the total amo	unt of points see	red)
			1.00)
		0	-6
+			
		0	-10
		v	-10
Final control			
		0	30
	specific task Module 1. T 1 2,5 The limit of t 2,5 e 3. Differentia 2,5 fodule 4. Resea 1 2,5 fodule 4. Resea 2,5 fodule 4. Resea 2,5 fodule 4. Resea 1 1 1 2,5	specific task of tasks per semester Module 1. The limit of th 1 10 2,5 3 The limit of the function. 1 10 2,5 3 e 3. Differentiability of the function. 1 10 2,5 3 fodule 4. Research of function. 1 10 2,5 3 fodule 4. Research of function. 1 10 2,5 3 3. Differentiability of the function. 1 10 2,5 3 And the function. 1 10 2,5 3 3. Differentiability of function. 1 10 2,5 3 3. Bonus points 3 Beducted from the total amount	specific task of tasks per semester minimum Module 1. The limit of the sequence. 0 1 10 0 2,5 3 0 The limit of the function. Continuity. 0 1 0 1 10 0 2,5 3 0 0 1 10 0 0 2,5 3 0 0 2,5 3 0 0 2,5 3 0 0 2,5 3 0 0 2,5 3 0 0 1 10 0 0 2,5 3 0 0 1 10 0 0 2,5 3 0 0 2,5 3 0 0 2,5 3 0 0 30nus points 0 0 0 Ideducted from the total amount of points sco 0 0

Rating – plan of the discipline

<u>A</u>nalysis I_____(the name of the discipline according to the working curriculum)

Direction 01.03.02 Applied Mathematics and Informatics Grade 1, semester 2

Rating-plan No. 2 (exam)

Types of educational activities	Points for a	Number	Ро	ints
of students	specific	of tasks	minimum	maximum
	task	per		
		semester		
	Module	1. Antiderivat	tive function	
Current control			0	10
Classroom work	1	10	0	10
Periodical control			0	10
Theoretical survey (inquiry)	2,5	4	0	10
	Module 2. Def	finite Integra	1	
Current control			0	15
Classroom work	1	15	0	15
Periodical control			0	10
Theoretical survey (inquiry)	2,5	4	0	10
	Module 3. Nu	merical series	S	
Current control			0	15
Classroom work	1	15	0	15
Periodical control			0	10
Theoretical survey (inquiry)	2,5	4	0	10
B	onus points			
1. Student academic competition or essay contest	_		0	5
2. Volunteering assistance in administrating of student academic competition or essay contest			0	5
Attendance (points are d	educted from	the total amo	ount of points sco	ored)
1. Attending lectures			0	-6
2. Attending practical				
classroom work (seminar,			0	-10
laboratory classes)				
	Final control			
Exam			0	30
Total points			45	100